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Physics
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Interior Ballistics

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BY

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THIRD EDITION

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PREFACE TO THE EDITION OF 1894

(SECOND EDITION)

WHEN, in the summer of 1889, it was decided by the Staff of the Artillery School to add to the curriculum a course of interior ballistics, the instructor of ballistics, knowing of no text-book on the subject in the English language entirely suited to the needs of the school, employed the time at his disposal before the arrival of the next class of student officers in studying up and arranging a course of instruction upon this subject, so important to the artillery officer. The text-book then planned was partially completed and printed on the Artillery School press, and has been tested by two classes of student officers.

In the summer of 1893 the author again had leisure to work on the unfinished text-book, but in the meantime he had found so much of it which admitted of improvement that, with the encouragement of Lieutenant-Colonel Frank, Second Artillery, the Commandant of the School, it was decided to rewrite nearly the entire work as well as to complete it according to the original plan by the addition of the last two chapters.

With the exception of portions of Chapters IV and V, the author claims no originality. He has simply culled from various sources what seemed to him desirable in an elementary text-book, arranged it all systematically from the same point of view and with a uniform notation.

ARTILLERY SCHOOL,
February 15, 1894.

PREFACE TO THE THIRD EDITION

THE second edition of this work was used as a text-book at the Artillery School until the School suspended operations at the outbreak of war with Spain, in April, 1898. This edition, having become exhausted, the author has been induced by the request of officers for whose wishes he has great respect, to prepare a new edition embodying the results of some investigations which were published in Volumes 24, 25 and 26 of the *Journal of the United States Artillery*, and which have been favorably received by Artillery Officers, both at home and abroad. The Journal articles have been rewritten, and many improvements attempted, suggested by friendly criticisms for which the author wishes to express his thanks.

As most of the formulas of interior ballistics in the present state of our knowledge of the subject are more or less empirical in their nature, many applications of the formulas deduced in Chapter IV are given in the following chapter to show their agreement with the results of actual firing with guns of widely different calibers. In this connection it is gratifying to be able to quote from an article published in the *Journal of the Royal Artillery*, vol. 36, No. 9, by Captain J. H. Hardcastle, R.A., who states, with reference to the formulas of Chapter IV as applied to firing-practice with English guns loaded with cordite, that "After many dozens of calculations I can find no serious disagreement between the results of calculation and experiment."

In this paper Captain Hardcastle has very ingeniously adapted the formulas of Chapter IV to slide-rule operations, thereby lessening the labor of calculation somewhat, though at the expense of accuracy in some cases. For the benefit of those who are accustomed to use the slide rule in making logarithmic computations a supplemental table of the X functions has been added to Table I, omitting the function X_5, which is not used in Captain Hardcastle's method.

This work was prepared primarily for the officers of our Coast Artillery Corps; but it is hoped that gun-designers and powder-manufacturers may find in it something useful to them.

The author desires to express his indebtedness to Lieut.-Colonel Ormand M. Lissak and Major Edward P. O'Hern of the Ordnance Department for valuable suggestions and for data employed in the "applications." Also to Captains Ennis and Bryant, for assistance in computing Table I.

PROVIDENCE, R. I.,
September 20, 1911.

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INTERIOR BALLISTICS

CHAPTER I

INTRODUCTION

Definition and Object.—Interior ballistics treats of the formation, temperature and volume of the gases into which the powder charge, in the chamber of a gun, is converted by combustion, and the work performed by the expansion of these gases upon the gun, carriage and projectile. Its object is the deduction and discussion of rules and formulas for calculating the velocity, both of translation and of rotation, which the gases of a given weight of powder of known composition and quality are able to impart to a projectile and their reaction upon the gun and carriage. The discussion of the formulas deduced will bring out many important questions, such as the proper relation of weight of charge to weight of projectile and length of bore, the best size and shape of the powder grains for different guns and their effect upon the maximum and muzzle pressures, the velocity of recoil, etc. The most approved formulas for calculating the pressures upon the surface of the bore will be given; but the methods which have been devised for building up the gun, so as best to resist these pressures, will not be entered upon here as their consideration belongs to another branch of the subject.

Early History of Interior Ballistics.—For more than five hundred years gunpowder—an intimate mixture of nitre, sulphur and charcoal,—was used almost exclusively as the pro-

pelling agent in firearms; and though it has been entirely superseded within the last quarter of a century by gun-cotton, nitro-glycerine, and their various compounds, yet it possessed many admirable qualities which the modern powders do not as yet so fully enjoy. It ignited easily without deflagration; its effects were regular and sure; its manufacture was economical, rapid and comparatively safe; it produced but little erosion in the bore. Finally, it kept well in transportation, and indefinitely in properly ventilated magazines. It is on record that experiments made with gunpowder, manufactured more than two centuries before, showed that it had lost none of its ballistic qualities. The principal objection to gunpowder, as compared to nitrocellulose powders, are the dense volumes of smoke accompanying its explosion, the fouling of the bore, and the comparatively large charges required to give the desired muzzle velocity, necessitating an abnormal enlargement of the powder chamber or an impracticable lengthening of the gun.

Robins' Experiments and Deductions.—The celebrated Benjamin Robins seems to have been the first investigator who had a tolerably correct idea of the circumstances relating to the action and force of fired gunpowder. In a paper which was read before the Royal Society in 1743 entitled, "New principles of gunnery," Robins described among other things some experiments he had made for determining the velocities of musket balls when fired with given charges of powder. These velocities were measured by means of the ballistic pendulum invented by Robins, "the idea of which is simply that the ball is discharged into a very large but movable block of wood, whose small velocity, in consequence of that blow, can be easily observed and accurately measured. Then, from this small velocity thus obtained, the large one of the ball is immediately derived from this simple proportion, viz., as the weight of the ball is to the sum of the weights of the ball and the block, so is the observed velocity of

the last to a fourth proportional, which is the velocity of the ball sought." *

The deductions which Robins makes from these experiments, so far as they relate to interior ballistics, may be summarized as follows:

(1) Gunpowder fired either in a vacuum or in air produces, by its combustion, a permanent elastic fluid or air.

(2) The pressure exerted by this fluid is, *cæteris paribus*, directly as its density.

(3) The elasticity of the fluid is increased by the heat it has at the time of explosion.

(4) The temperature of the fluid at the moment of combustion is at least equal to that of red-hot iron.

(5) The maximum pressure exerted by the fluid is equal to about 1,000 atmospheres.

(6) The weight of the permanent elastic fluid disengaged by the combustion is about three-tenths that of the powder, and its volume at ordinary atmospheric temperature and pressure is about 240 times that occupied by the charge.

These deductions, considering the extremely erroneous and often absurd opinions that were entertained by those who thought upon the subject at all in Robins' time—and even down to the close of the century—show that Robins is well entitled to be called the "father of modern gunnery."

Hutton's Experiments.—Dr. Charles Hutton, professor of mathematics in the Royal Military Academy, Woolwich, continued Robins' experiments at intervals from 1773 to 1791. He improved and greatly enlarged the ballistic pendulum so that it could receive the impact of 1-pound balls, whereas that used by Robins was adapted for musket balls only. Hutton's experiments are given in detail in his thirty-fourth, thirty-fifth, thirty-sixth, and thirty-seventh tracts. They verify most of

* Hutton's "Mathematical Tracts," vol. 3, p. 210 (Tract 37), London, 1812.

Robins' deductions, but with regard to Robins' estimate of the temperature of combustion and the maximum pressure Hutton says: "This was merely guessing at the degree of heat in the inflamed fluid, and, consequently, of its first strength, both which in fact are found to be much greater." * His own estimate of the temperature is double that of Robins, and he places the maximum pressure of fired gunpowder at 2,000 atmospheres. Hutton gives a formula for the velocity of a spherical projectile at any point of the bore, upon the assumption that the combustion of the charge is instantaneous and that the expansion of the gas follows Mariotte's law—no account being taken of the loss of heat due to work performed—a principle which at that time was unknown.

D'Arcy's Method.—In 1760 the chevalier D'Arcy sought to determine the law of pressure of the gas in the bore of a musket by measuring the velocity of the projectile at different points of the bore. This he accomplished by successively shortening the length of the barrel and measuring for each length the velocity of the bullet by means of a ballistic pendulum. Having obtained from these experiments the velocities of the bullets for several different lengths of travel, the corresponding accelerations could be calculated, and then the pressures, by multiplying the accelerations by the mass. This was the first attempt to determine the law of pressures dynamically.

Rumford's Experiments with Fired Gunpowder.—The first attempt to measure directly the pressure of fired gunpowder was made, in 1792, by our countryman, the celebrated Count Rumford. A most interesting account of his experiments is given in his memoir entitled "Experiments to determine the force of fired gunpowder," † which must be regarded as the most important contribution to interior ballistics which had been

* Tracts, vol. 3, p. 211.

† Philosophical Transactions, London, 1797, p. 222; also "The Complete Works of Count Rumford," Boston, 1870, vol. 1, p. 98.

made up to that time. The apparatus used by Rumford consisted of a small and very strong wrought-iron mortar (or eprouvette), which rested with its axis vertical upon a solid stone foundation. This mortar (or barrel, as Rumford calls it), was 2.78 inches long and 2.82 inches in diameter at its lower extremity and tapered slightly toward the muzzle. The bore (or chamber) was cylindrical, one-fourth of an inch in diameter and 2.13 inches deep. At the centre of the bottom of the barrel there was a projection 0.45 inch in diameter and 1.3 inches long, having an axial bore 0.07 inch in diameter connecting with the chamber above, but closed below, forming a sort of vent, but having no opening outside.

By this arrangement the charge could be fired without any loss of gas through the vent by the application of a red-hot ball provided with a hole, into which the projecting vent-tube could be inserted, which latter would thus become in a short time sufficiently heated to ignite the powder. The upper part of the bore or muzzle was closed by a stopper made of compact, well-greased sole leather, which was forced into the bore, until its upper surface was flush with the face of the mortar, and upon this was placed the plane surface of a solid hemisphere of hardened steel, whose diameter was 1.16 inches. "Upon this hemisphere the weight made use of for confining the elastic fluid generated from the powder in its combustion reposed. This weight in all the experiments, except those which were made with very small charges of powder, was a piece of ordnance of greater or less dimensions or greater or less weight, according to the force of the charge, placed vertically upon its cascabel upon the steel hemisphere which closed the end of the barrel; and the same piece of ordnance, by having its bore filled by a greater or smaller number of bullets, as the occasion required, was made to serve for several experiments." *

* Rumford's Works, vol. 1, p. 121.

As one of the objects of Rumford's experiments was to determine the relation between the pressure of the powder gases and their density, he varied the charge, beginning with 1 grain, and for each charge placed a weight, which he judged was about equivalent to the resulting pressure, upon the hemisphere. If, on firing, the weight was lifted sufficiently to allow the gases to escape, it was increased for another equal charge; and this was repeated until a weight was found just sufficient to retain the gaseous products—that is, so that the leathern stopper would not be thrown out of the bore, but only slightly lifted. The density of the powder gases could easily be determined by comparing the weight of the charge with the weight of powder required to completely fill the chamber and vent, which latter was about 25½ grains troy. Rumford increased the charges a grain at a time from 1 grain to 18 grains, and from a mean of all the observed pressures he deduced the empirical formula,

$$p = 1.841x^{1+.0004x},$$

in which p is the pressure in atmospheres and x the density of loading to a scale of 1000—that is, for a full chamber $x = 1000$; for one-half full $x = 500$, and so on. This formula gives 29,178 atmospheres for the maximum pressure—that is, when the powder entirely fills the space in which it is fired. In this case the value of x is 1000, and Rumford's pressure formula becomes

$$p = 1.841 \times 1000^{1.4} = 29178$$

Nearly a century later Noble and Abel (see Chapter II) found by their experiments, which are entirely similar in character to those of Rumford, that the maximum pressure of fired gunpowder is but 6,554 atmospheres, or 43 tons per square inch; and this result has been accepted by all writers on interior

ballistics as being very near the truth. Their formula for the pressure in terms of Rumford's x is

$$p = \frac{2.818x}{1 - 0.00057x}$$

in which p and x are defined as before. If in this formula we make $x = 1000$, we have, as already stated,

$$p = \frac{2.818 \times 1000}{1 - 0.57} = 6554$$

For small densities of loading, Noble and Abel's formula gives greater pressures than Rumford's principally because the powder used by the later investigators was the stronger; but as the densities increase this is reversed. With a charge of 18 grains, for which $x = 702$, Noble and Abel's formula gives a pressure of 3,298 atmospheres, while Rumford's gives 8,140 atmospheres. To enable us to understand the cause of this great difference in the results obtained by these eminent savants (which is very instructive), we will go a little into detail. Two experiments were made by Rumford with a charge of 18 grains of powder. In the first of these a 24-pounder gun, weighing 8,081 pounds, was placed vertically on its cascabel upon the steel hemisphere closing the muzzle of the barrel. When the charge was fired "the weight was raised with a very sharp report, louder than that of a well-loaded musket." The barrel was again loaded with 18 grains as before, and enough shot were placed in the bore of the 24-pounder gun to increase its weight to 8,700 pounds. Upon firing the powder by applying the red-hot ball "the vent-tube of the barrel was burst, the explosion being attended with a very loud report." These experiments were the eighty-fourth and eighty-fifth, and closed the series.

In the eighty-fourth experiment a weight of 8,081 pounds was actually raised by the explosion of 18 grains of powder (about one-fourth the service charge of the Springfield rifle),

acting upon a circular area one-quarter of an inch in diameter. To raise this weight under the circumstances would require a pressure of more than 11,200 atmospheres, while, as we have seen, the actual pressure due to this density of loading, according to Noble and Abel's formula, is but 3,298 atmospheres. Evidently then the weight in this experiment was not raised by mere pressure; but we must attribute a great part of the observed effect (in consequence of the position of the charge at the bottom of the bore) to the energy with which the products of combustion impinged against the leathern stopper, which had only to be raised 0.13 inch (the thickness of the leather) to allow the gases to escape. In Noble and Abel's experiments there was no such blow from the products of combustion because the apparatus for determining the pressure (crusher gauge) was placed within the charge. Had the leathern stopper in Rumford's experiments been a little longer, it is probable that his conclusions would have been entirely different.

Rodman's Inventions and Experiments.—We have space only to mention the names of Gay-Lussac, Chevreul, Graham, Piobert, Cavalli, Mayevski, Otto, Neumann, and others, who did original work, of more or less value, for the science of interior ballistics prior to the year 1860. We will, however, dwell a few moments on the important work done by Captain (afterwards General) T. J. Rodman, of our own Ordnance Department, between the years 1857 and 1861.* The objects of Rodman's experiments were: First, to ascertain the pressure exerted upon different points of the bore of a 42-pounder gun in firing under various circumstances. Second, to determine the pressures in the 7-inch, 9-inch, and 11-inch guns when the charges of powder and the weight of projectiles were so proportioned that there should be the same weight of powder behind, and

* "Experiments on Metal and Cannon and Qualities of Cannon Powder," by Captain T. J. Rodman, Boston, 1861.

the same weight of metal in front of each square inch of area of cross-section of the bore. Third, to determine the differences in pressure and muzzle velocity due to the variations in the size of the powder grains; and, fourth, to determine the pressures exerted by gunpowder burned in a close vessel for different densities of loading.

For the purpose of carrying out these experiments Rodman, instead of using the system of varying weights employed by Rumford, invented what he called the "indenting apparatus," which has since been extensively used, not only in this country but in all foreign countries, under the name of Rodman's pressure (or cutter) gauge; and which is too well known to require a description.

The maximum pressure of gunpowder when exploded in its own space, as determined by Rodman by the bursting of shells filled with powder, ranged from 4,900 to 12,600 atmospheres; the mean of all the experiments giving 8,070 atmospheres, or 53 tons per square inch. These results, though much nearer the truth than those deduced by Rumford, are still about 25 per cent. greater than Noble and Abel's deductions; and this is undoubtedly due to the position of the pressure gauge, which was placed near the exterior surface of the shell, so that when the products of combustion had reached the gauge they had acquired a considerable energy which probably exaggerated the real pressure. The same causes, it will be remembered, vitiated Rumford's experiments. In both cases it was as if a charge of small shot had been fired with great velocity against the leathern stopper in the one case, or the end of the piston of the indenting tool in the other.

General Rodman was the first person to suggest the proper shape for powder grains, in order to diminish the initial velocity of emission of gas and to more nearly equalize the pressure in the bore of the gun. For this purpose he employed what he termed a "perforated cake cartridge" composed of disks of

compressed powder from 1 to 2 inches thick and of a diameter to fit the bore. Rodman demonstrated that such a form of cartridge would relieve the initial strain by exposing a minimum surface at the beginning of combustion, while a greater volume of gas would be evolved from the increasing surfaces of the cylindrical perforations as the space behind the projectile became greater; and this would tend to distribute the pressure more uniformly along the bore. Rodman's experiments with this powder in the 15-inch cast-iron gun which he had recently constructed for the government—and which is without doubt the most effective and the best smooth-bore gun ever made—fully confirmed his theory; but for many reasons he found it more convenient and equally satisfactory to build up the charge by layers of pierced hexagonal prisms about an inch in diameter fitting closely to one another, instead of having them of the diameter of the bore.

The war of the rebellion which was inaugurated while General Rodman was in the midst of his discoveries and inventions, put an end forever to his investigations, but his ideas were speedily adopted in Europe, and his "prismatic powders," but slightly modified, are extensively used.

Modern Explosives.—Gun-cotton, made by immersing cleaned and dried cotton waste in a mixture of strong nitric and sulphuric acids, was discovered by Schönbein of Basel, in 1846, who immediately proposed to employ it as a substitute for gunpowder. General von Lenk made many experiments with gun-cotton by compressing it into cubes or cylinders, with the idea of employing it for artillery use. But all his efforts failed from the fact that, no matter how much it was compressed, it was still mechanically porous; and when ignited in a gun the flame and hot gases speedily penetrated the mass causing it to detonate, or, at least, to approach dangerously near to detonation. It was not until the discovery in the early eighties that gun-cotton could be dissolved or made into a paste, or colloid,

by acetone and other so-called solvents, that it was possible to employ it as a propellant. In this condition, when moulded into grains and thoroughly dried, it loses its mechanical porosity and burns from the surface in parallel layers, the grain retaining its original form until completely consumed.

Gun-cotton is mixed in certain proportions with nitro-glycerine to form nearly all the powders employed for war purposes. For example, the powder used in the British army and navy (called cordite), consists of 65 per cent. of gun-cotton, 30 per cent. of nitro-glycerine and 5 per cent. of mineral jelly or vaseline, this latter being used as a preservative. This is also very nearly the composition of the powder used in the United States army and navy. For a full account of the properties, manufacture and uses of gun-cotton and nitro-glycerine, the reader is referred to General Weaver's "Notes on Explosives."

Density of Powder.—By density of a powder is meant its specific gravity, or the ratio of the weight of a given volume of the powder to the weight of an equal volume of water at the standard temperature. It is sometimes referred to as *mercurial density*, since it may be determined by an apparatus which utilizes the property of mercury of filling the interstices between the grains without penetrating into the pores or uniting chemically with the powder. The density varies somewhat according to the pressure to which the grains were subjected during the manufacture and ranges from about 1.56 to 1.65.

Inflammation and Combustion of a Grain of Powder.—Inflammation is the spreading of the flame over the free surface of the grain from the point of ignition. Combustion is the propagation of the burning into the interior of the grain. Ignition is produced by the sudden elevation of the temperature of a small portion of the grain to about 180° C. (in the case of cordite) either by contact with an ignited body, by mechanical shock or friction, or by detonation of a fulminate. The velocity

of inflammation depends upon the nature of the source of heat which ignites it, upon the state of the surface of the grain and upon its density and dryness. The combustion of a grain takes place in successive concentric layers, and in free air equal thicknesses are burned in equal times. As the mass of gas disengaged in any given time is proportional to the quantity of powder burned during the same time, and, therefore, proportional to the surface of inflammation, it follows that the emission of gas is largely influenced by the form of the grain. For example, if the grain is spherical the surface of inflammation decreases rapidly up to the end of its burning where it is zero. On the other hand, the surface of inflammation (or of combustion) of a multi-perforated grain increases until it is nearly consumed.

Inflammation and Combustion of a Charge of Powder.— The inflammation of a charge of powder involves the transmission of the flame from one grain to another. Its velocity depends not only upon the inflammability of the grain but also upon the facility with which the gases first generated are able to penetrate the charge. This is assisted by a proper arrangement of the grains composing the charge and also by placing an igniter of fine rifle powder at each end of the cartridge. The combustion of a charge composed of grains of the same form and dimensions should, from what has been said, practically terminate at the same time with each or any grain; and, therefore, the time of combustion of a charge increases with the size of the grains, and is in all cases with service powders much longer than the time of inflammation.

If a charge of powder be confined in a close vessel and ignited, its combustion takes place silently, and permanent gases and a certain amount of solid matter are produced which can be collected for analysis by opening the vessel, as in the experiments of Noble and Abel described in Chapter II. In this case no work is performed by the gases, and the accompanying phe-

nomena are comparatively simple. But if the combustion takes place in a chamber of which one of the walls is capable of moving under the tension of the gases, which condition is realized in cannon, the resulting phenomena are much more complicated, as a little consideration will show.

When the charge of powder in the chamber of a loaded gun is ignited at both ends of the cartridge, all the grains will be inflamed practically simultaneously. The first gaseous products formed will expand into the air-spaces of the chamber and almost immediately acquire a tension sufficient to start and overcome the forcing of the projectile. This latter once in motion will encounter no resistances in the bore comparable with those which opposed its start, and its velocity will rapidly increase under the continued action of the pressure of the gases. This pressure will also increase at first; for, though the displacement of the projectile gives a greater space for the expanding gases, this is more than compensated for by a more abundant disengagement of gas. But the pressure soon reaches its maximum; for if, on the one hand, the disengagement of gas is accelerated by the increase of pressure, on the other hand the increasing velocity of the projectile offers more and more space for the gases to expand in. The velocity itself would soon reach a maximum if the bore were sufficiently prolonged; for in addition to the friction and the resistance of the air, both of which retard the motion of the projectile, the propulsive force decreases by the expansion and cooling of the gases. Therefore the retarding forces will in time predominate and the projectile be brought to rest. Its velocity starting from zero passes to its maximum and if the gun terminated at this point the projectile would leave the bore with the greatest velocity the charge was capable of communicating to it.

So far only charges in general have been considered. Take, now, a charge composed of small grains of slight density. The initial surface of inflammation will be very great and the emission

of gas correspondingly abundant. The pressure will increase rapidly, and, in consequence, the velocity of combustion. It results from this that the grains will be consumed nearly as soon as inflamed, and this before the projectile has had time to be displaced by a sensible amount. Hence all the gases of the charge, disengaged almost instantaneously, will be confined an instant within the chamber; their tension will be very great, and they will exert upon the walls of the gun a sudden and violent force which may rupture the metal, and which in all cases will produce upon the gun and carriage shocks which are destructive to the system and prejudicial to accuracy of fire. On the other hand the projectile will be thrown quickly forward, as by a blow from a hammer.

If, on the contrary, the charge is made up of large grains of great density, the total weight of gas emitted will be the same as before; but the mode of emission will be different. The initial surface of inflammation will be less, and the initial tension of the gas not so great. The combustion will take place more slowly, and will be only partially completed when the projectile shall have begun to move. The pressure of the gases will attain a maximum less than in the preceding case, but the pressure will decrease more slowly. Under the continued action of this pressure, the velocity of the projectile will be rapidly accelerated and at the muzzle will differ but little from that obtained by the fine powder, without producing upon the gun and carriage the destructive effects mentioned above.

CHAPTER II

PROPERTIES OF PERFECT GASES

Mariotte's Law.—When a mass of gas is subjected to pressure the volume diminishes until the increased tension just balances the pressure; and it was found by experiment that if the temperature of the gas remains constant, the tension, or pressure, is inversely proportional to the volume. Thus, if v_1 and v_2 represent different volumes of the same mass of gas and p_1 and p_2 the corresponding tensions, or pressures, then if the temperature is the same for both volumes we have the proportion:

$$v_1 : v_2 = \frac{1}{p_1} : \frac{1}{p_2}$$

Hence

$$v_1 p_1 = v_2 p_2 = \text{constant.}$$

That is, for every mass of gas at invariable temperature the product of the volume and tension is constant. This law is generally called Mariotte's law, though it was first discovered by the English chemist Robert Boyle, in 1662, and verified by Mariotte in 1679.

Specific Volume.—The specific volume of a gas is the volume of unit weight at zero temperature and under the normal atmospheric pressure. Designate the specific volume by v_o and the normal atmospheric pressure by p_o. Then we have by Mariotte's law

$$v p = v_o p_o$$

Specific Weight.—The specific weight of a gas is the weight of unit volume at zero temperature and under the pressure p_o. It is therefore the reciprocal of the specific volume v_o.

Law of Gay-Lussac.—The coefficient of expansion of a gas is the same for all gases, and is sensibly constant for all temperatures and pressures. Let, as before, v_o be the specific volume, v_t the volume at temperature t and α the coefficient of expansion. Then the variation of volume by Gay-Lussac's law will be expressed by the equation

$$v_t - v_o = \alpha\, t\, v_o;$$

whence

$$v_t = v_o\,(1 + \alpha\, t)$$

The value of the coefficient α is approximately $\frac{1}{273}$ for each degree centigrade. The last equation may, therefore, be written

$$v_t = v_o\left(1 + \frac{t}{273}\right)$$

Characteristic Equation of the Gaseous State.—The last equation, which expresses Gay-Lussac's law, may be combined with Mariotte's law, introducing the pressure p. The problem may be enunciated as follows: Having given the specific volume of a gas v_o to determine its volume v_t at a temperature t under the corresponding pressure p_t.

Let x be the volume v_t would become at 0° $C.$, under the pressure p_t. Then by Gay-Lussac's law

$$v_t = x\,(1 + \alpha\, t)$$

and by Mariotte's law

$$p_t\, x = p_o\, v_o;$$

whence eliminating x,

$$p_t\, v_t = p_o\, v_o\,(1 + \alpha\, t) = \frac{p_o\, v_o}{273}\,(273 + t)$$

Since $\frac{p_o\, v_o}{273}$ is constant, put

$$R = \frac{p_o\, v_o}{273};$$

whence

$$p_t v_t = R\,(273 + t);$$

or, dropping the subscripts as no longer necessary,

$$p\,v = R\,(273 + t)$$

The temperature $(273 + t)$ is called the absolute temperature, and is reckoned from a zero placed 273 degrees below the zero of the centigrade scale. Calling the absolute temperature T there results finally

$$p\,v = R\,T \qquad \qquad (1)$$

which is called the characteristic equation of the gaseous state. It is simply another expression of Mariotte's law in which the temperature of the gas is introduced.

Equation (1) expresses the relation existing between the pressure, volume and absolute temperature of a unit weight of gas. For any number y units of weight occupying the same volume v the relation evidently becomes

$$p\,v = y\,R\,T \qquad \qquad (2)$$

A gas supposed to obey exactly the law expressed in equation (1) is called a perfect gas, or is said to be theoretically in the perfectly gaseous state. This condition represents an ideal toward which gases approach more nearly as their state of rarefaction increases. Of all gases, hydrogen approximates most closely to such an hypothetical substance, though at ordinary temperatures the simple gases, nitrogen, oxygen and atmospheric air, may for most practical purposes be considered perfect gases.

Thermal Unit.—The heat required to raise the temperature of unit weight of water at the freezing point one degree of the thermometer is called a thermal unit. There are two thermal units in general use, namely: the British thermal unit (B. T. U.), which is the heat required to raise the temperature of one pound

of water from 32° F. to 33° F.; and the French thermal unit (called a calorie), which is the heat required to raise the temperature of one kilogram of water from 0° C. to 1° C. There is still another thermal unit of frequent use, namely: the heat required to raise the temperature of one pound of water from 0° C. to 1° C., and which may be designated as the pound-centigrade (P. C.) unit.

Mechanical Equivalent of Heat.—The mechanical equivalent of heat is the work equivalent of a thermal unit, and will be designated by E. According to Rowland the value of E is 778 foot-pounds for a B. T. U. Since a degree of the centigrade scale is $\frac{9}{5}$ of a degree of the Fahrenheit scale, we have for a P. C. thermal unit, $E = \frac{9}{5} \times 778 = 1400.4$ foot-pounds. Also since there are 3.280869 feet in a metre, the value of E for a calorie is

$$\frac{1400.4}{3.280869} = 426.84 \text{ kilogram-metres.}$$

Specific Heat.—The quantity of heat, expressed in thermal units, which must be imparted to a unit weight of any substance to increase its temperature one degree of the thermometer, or the quantity of heat given up by the substance while its temperature is lowered one degree, is called its specific heat. The specific heat of different substances varies greatly. Thus, if a pound of mercury and a pound of water receive the same quantity of heat the temperature of the former will be much greater that the latter. Indeed, it requires about 32 times as much heat to raise the temperature of water 1° as it does to raise the temperature of mercury by the same amount.

The heat imparted to a substance is expended in three different ways: 1. Increasing the temperature, which may be called vibration work; 2. In doing internal or disgregation work; 3. In doing external work by expansion. If it were

possible to eliminate the two latter, we should get the true specific heat, or the heat necessary to increase the temperature simply. For a perfect gas, however, the disgregation work is zero, and for all substances the disgregation work is small in comparison with the vibration work. The specific heat of a gas may be determined in two different ways, giving results which are of fundamental importance in thermodynamics, namely: Specific heat under constant pressure, and specific heat under constant volume.

Specific Heat of a Gas Under Constant Pressure.—To fix the ideas suppose a unit weight of gas to be confined in a spherical envelope capable of expanding without the expenditure of work and which allows no heat the gas may have to escape, and to be in equilibrium with the constant pressure of the atmosphere. Under these conditions let a certain quantity of heat be applied to the gas just sufficient to raise its temperature one degree of the thermometer after it has expanded until equilibrium is again restored. This quantity of heat, in thermal units (designated by C_p), is called specific heat under constant pressure.

Specific Heat Under Constant Volume.—Next repeat the experiment just described, but replacing the elastic envelope, which by hypothesis permitted the gas to expand freely, by a rigid envelope, thus keeping the volume of the gas constant while heat is applied. It will now be found that there will less heat be required to raise the temperature of the gas one degree. The quantity of heat required in this case is called the specific heat under constant volume, and in terms of the thermal unit employed, is designated by C_v.

The number of molecules of gas being the same in both experiments and the temperatures being equal, it follows that the quantity of heat absorbed by the gas, or the vibration work, is the same in both experiments. But in the experiment made under constant volume the heat absorbed is necessarily equal

to the total heat supplied, namely, C_v thermal units, since the envelope is considered impermeable to heat. Therefore in the first experiment there is a loss of heat equal to $C_p - C_v$ thermal units. This last heat must then have been expended in overcoming the atmospheric pressure in expanding; and the work done will be found by multiplying $C_p - C_v$ by the mechanical equivalent of heat. That is, for an increase of one degree of temperature,

$$\text{Work of expansion} = (C_p - C_v)\,E.$$

The work of overcoming a constant resistance is measured by the product of the resistance into the path described. In the case of the expanding gas just considered the constant resistance is the atmospheric resistance p_o; and the path described is measured by the increase of volume of the gas. To determine this latter Gay-Lussac's law gives for the centigrade scale

$$v_t - v_o = \frac{t\,v_o}{273}$$

and therefore for an increase of temperature of one degree there is an increase of volume equal to $v_o/273$. The work of expansion for one degree is, therefore,

$$\frac{p_o\,v_o}{273} = R.$$

The quantity R is, then, the external work of expansion performed under atmospheric pressure by unit weight of gas when its temperature is raised one degree centigrade. But this work of expansion has already been found equal to $(C_p - C_v)$ E. There results, therefore, the important equation

$$(C_p - C_v)\,E = \frac{p_o\,v_o}{273} = R \quad . \quad . \quad . \quad . \quad (3)$$

for the centigrade scale of temperature. For the Fahrenheit scale the equation becomes

$$(C_p - C_v)\,E = \frac{p_o\,v_o}{491.4} \quad . \quad . \quad . \quad . \quad . \quad (3^1)$$

Numerical Value of R.—The numerical value of R for any particular gas depends upon the units of length and weight adopted, the atmospheric pressure, the specific weight of the gas and the scale of temperature. Throughout this chapter the foot and pound will be employed for the units of length and weight, respectively; and generally the centigrade scale of temperature will be used. The adopted value of the atmospheric pressure is

$$p_o = 10333 \text{ kgs. per m.}^2 \quad \log = 4.01423.$$
$$p_o = 2116.3 \text{ lbs. per ft.}^2 \quad \log = 3.32558.$$
$$p_o = 14.6967 \text{ lbs. per in.}^2 \quad \log = 1.16722.$$

As an example, find the numerical value of R for atmospheric air. The specific weight of this gas, according to the best authorities, is 0.080704 lbs. The specific volume is the reciprocal of this; or $V_o = 12.3909$ c. ft. Therefore,

$$R = \frac{2116.3 \times 12.3909}{273} = 96.056 \text{ foot-pounds.}$$

Therefore, for one pound of this gas,

$$p\,v = 96.056\,T;$$

and for y pounds

$$p\,v = 96.056\,y\,T.$$

Law of Dulong and Petit.—*The product of the specific heat of a perfect gas under constant volume, by its density, is a constant number.*

By the density of a gas is meant its specific weight expressed in terms of the specific weight of atmospheric air taken as unity. If C_{va} is the specific heat of air at constant volume and C_v and d the specific heat at constant volume, and density, respectively, of any other gas, then in accordance with this law,

$$C_v\,d = C_{va}.$$

Determination of Specific Heats.—The specific volume and the specific heat at constant pressure of a gas can both be

determined with great accuracy by experiment; but the specific heat under constant volume is almost impossible to measure directly on account of the dissipation of heat through the sides of the vessel containing the gas. It can, however, be computed by equation (3) which gives

$$C_v = C_p - \frac{R}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By means of this equation and the direct determination of specific heats under constant pressure, Regnault has deduced the following law for perfect gases:

The specific heats under constant pressure and constant volume are independent of the pressure and volume.

The following table gives the specific weights, volumes and heats of those gases which approximate most nearly to the theoretically perfect gas. The values of R were computed by (1) and those of C_v by (4). The temperature is supposed to be 0° C., and the barometer to stand at 760 mm. = 29.922 in.:

Gas	Specific Weight	Specific Volume	R	C_p	C_v
	Pounds	Cubic Feet			
Atmospheric air	0.080704	12.3909	96.056	0.23751	0.16892
Nitrogen	0.078394	12.7569	98.887	0.24380	0.17319
Oxygen..........	0.089230	11.2070	86.878	0.21751	0.15547
Hydrogen.......	0.005590	178.8910	1386.8	3.40900	2.41873

Ratio of Specific Heats.—In the study of interior ballistics the values of C_p and C_v for the gases given off by the explosion of the charge are of little importance. It suffices generally to know their ratio which is constant for perfect gases and approximately so for all gases at the high temperature of explosion. That this ratio is constant for perfect gases may be shown as follows: Since

$$R = \frac{p_o v_o}{273} = \frac{p_o}{273\, w_o} = \frac{p_o}{273\, d\, w_a}$$

in which w_a is the specific weight of atmospheric air, we shall have for two gases distinguished by accents, the relation

$$\frac{R'}{R''} = \frac{d''}{d'}\,;$$

that is, the values of R for two perfect gases are inversely as their densities. But by the law of Dulong and Petit we have

$$\frac{C'_v}{C''_v} = \frac{d''}{d'} = \frac{R'}{R''} \text{ (as shown above).}$$

Therefore

$$\frac{R'}{C'_v} = \frac{R''}{C''_v} = \text{constant.}$$

Therefore from equation (4),

$$\frac{C_p}{C_v} = 1 + \frac{R}{C_v E} = \text{constant} = n \text{ (say).}$$

If we compute n by means of atmospheric air, we shall have

$$n = 1 + \frac{96.056}{0.16892 \times 1400.4} = 1.406.$$

Relations Between Heat and Work in the Expansion of Perfect Gases.—The relations which exist between the variations of the volume and pressure of a given weight of gas and the heat necessary to produce them, may now be determined from equation (1) as follows: This equation is

$$p\,v = R\,T$$

and contains three arbitrary variables p, v and T. If we suppose an element of heat, $d\,q$, to be applied to the gas, the temperature will generally be augmented by an elementary amount $d\,T$, and this may be accomplished in three different ways:

1. The volume may increase by the element $d\,v$ without altering the pressure. 2. The pressure may increase by $d\,p$

while the volume remains constant. 3. The volume and pressure may both vary at the same time. We will consider each of these cases separately.

1. Differentiating (1), supposing p constant, we have

$$d\,T = \frac{p\,d\,v}{R};$$

and therefore the quantity of heat communicated to the gas will be, in thermal units, from the definition of specific heat,

$$d\,q = C_p\,d\,T = \frac{C_p\,p\,d\,v}{R}$$

2. If, the volume v remaining constant, the pressure is varied by $d\,p$, we shall have, proceeding as before,

$$d\,q = C_v\,d\,T = \frac{C_v\,v\,d\,p}{R}$$

3. If the volume and pressure vary together, the corresponding element of heat will be the sum of the partial variations given above. That is

$$d\,q = \frac{1}{R}\,(C_p\,p\,d\,v + C_v\,v\,d\,p) \quad . \quad . \quad . \quad (5)$$

The differential of (1) is

$$R\,d\,T = p\,d\,v + v\,d\,p; \quad . \quad . \quad . \quad . \quad (6)$$

whence, eliminating $v\,d\,p$ between (5) and (6), there results

$$d\,q = C_v\,d\,T + \frac{C_p - C_v}{R}\,p\,d\,v \quad . \quad . \quad . \quad (7)$$

Whence, since C_p, C_v and R are constants for the same gas,

$$q = C_v \int d\,T + \frac{C_p - C_v}{R} \int p\,d\,v.$$

The first integral represents the change of temperature and the second the external work of expansion. Denoting by T_1 and T the initial and final temperatures of the expanding gas and by W the external work, we have

$$q = (T_1 - T)\,C_v + \frac{C_p - C_v}{R}\,W \quad . \quad . \quad . \quad (8)$$

Isothermal Expansion.—If we suppose the initial temperature T_1 to remain constant, that is, that just sufficient heat is imparted to the gas while it expands to maintain its initial temperature, equation (8) becomes

$$q = \frac{C_p - C_v}{R} W.$$

We see in this case that the quantity of heat absorbed by the gas is proportional to the external work done. The quantity $\frac{R}{C_p - C_v}$ is, therefore, the ratio of the effective work of a unit weight of gas to the quantity of heat absorbed, or the mechanical equivalent of heat, E. Therefore

$$E = \frac{R}{C_p - C_v}$$

a result already established by another method.

The work performed, therefore, by the isothermal expansion of unit weight of gas is given by the equation

$$W = E\,q = 1400.4\,q \text{ foot-pounds}, \quad . \quad . \quad . \quad (9)$$

where q is expressed in P. C. thermal units.

The work of an isothermal expansion may also be expressed in terms of the initial and final volumes or pressures. Thus, substituting in the general equation of the work of expansion,

$$W = \int p\,d\,v,$$

the value of p from (1) and integrating between the limits v_1 and v, we have

$$W = R\,T_1 \log_e \frac{v}{v_1} = p_1\,v_1 \log_e \frac{v}{v_1} \quad . \quad . \quad . \quad (10)$$

where v is the greater volume and v_1 the less.

Since from (1)

$$\frac{v}{v_1} = \frac{p_1}{p}$$

we also have

$$W = p_1 v_1 \log_e \frac{p_1}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

in which p_1 is the greater tension and p the less.

The reciprocal of E may be called the heat equivalent of work, that is, the quantity of heat equivalent to a unit of work. Therefore from (9), (10) and (11), we have

$$\left.\begin{aligned} q = \frac{W}{E} &= \frac{p_1 v_1}{E} \log_e \frac{v}{v_1} \\ &= \frac{p_1 v_1}{E} \log_e \frac{p_1}{p} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Equations (10) and (11), by inverting the ratios of volumes or pressures, evidently hold good when the initial volume v_1 and initial tension p_1 are changed by compression under constant temperature into the less volume v and greater tension p.

Adiabatic Expansion.—If a gas expands and performs work in an envelope impermeable to heat, so that it neither receives nor gives up heat during the expansion, the transformation is said to be adiabatic. In such an expansion the temperature and tension of the gas both diminish and the work performed must be less than for an isothermal expansion, other things being equal. For an adiabatic expansion, q is zero in (8) and, therefore, since the temperature diminishes,

$$\left.\begin{aligned} W &= \frac{R\, C_v}{C_p - C_v} (T_1 - T) \\ &= \frac{R}{n - 1} (T_1 - T) \\ &= C_v E\, (T_1 - T) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Therefore in an adiabatic expansion the work done is proportional to the fall of temperature.

Next consider equation (7), where, if we make $d\,q$ zero, it becomes

$$0 = C_v\, d\,T + \frac{C_p - C_v}{R}\, p\, d\,v;$$

which, by dividing by C_v and replacing p by its value $\frac{RT}{v}$, reduces to

$$o = \frac{dT}{T} + (n - 1)\frac{dv}{v}$$

Integrating between limits, we have

$$\frac{T}{T_1} = \left(\frac{v_1}{v}\right)^{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Again, making dq zero in (5), we have

$$o = C_p\, p\, dv + C_v\, v\, dp,$$

which may be written (dividing by $C_v\, p\, v$)

$$o = n\,\frac{dv}{v} + \frac{dp}{p}$$

Integrating between limits, we have

$$\left(\frac{v_1}{v}\right)^n = \frac{p}{p_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Combining (14) and (15) gives the important relations

$$\left(\frac{v_1}{v}\right)^{n-1} = \frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{n-1}{n}} \quad . \quad . \quad . \quad . \quad (16)$$

By means of (16) the work of an adiabatic expansion given by (13) may be expressed either in terms of the initial and terminal volumes, or of the initial and terminal pressures. Thus, since

$$T_1 - T = T_1\left(1 - \frac{T}{T_1}\right)$$

the last of equations (13) may be written,

$$\left.\begin{aligned} W &= C_v E T_1 \left\{ 1 - \left(\frac{v_1}{v}\right)^{n-1} \right\} \\ &= C_v E T_1 \left\{ 1 - \left(\frac{p}{p_1}\right)^{\frac{n-1}{n}} \right\} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (17)$$

Examples.

1. Determine the volume of 5 pounds of oxygen at a pressure of 50 pounds per square inch by the gauge, and at a temperature of 60° C.

The real pressure is gauge pressure plus the atmospheric pressure = 50 + 14.6967 = 64.6967 lbs. per in.^{2} Therefore, $p = 144 \times 64.6967$ lbs. per ft.^{2} $T = 273 + 60 = 333°$. $R =$ 86.878. Therefore from (2),

$$v = \frac{5 \times 86.878 \times 333}{144 \times 64.6967} = 15.527 \text{ ft.}^3$$

2. One pound of atmospheric air occupying a volume of one cubic foot has a tension of 50,000 lbs. per ft.^{2} What is its temperature by the Fahrenheit scale?

For the centigrade scale we have $R = 96.056$, $v = 1$, $p =$ 50,000 and $y = 1$.

$$\text{Therefore } T = \frac{50000}{96.056} = 520.°54 \text{ C.} = 968.°97 \text{ F.}$$

$$\therefore t = 968.97 - 491.4 = 477.°57 \text{ F.}$$

3. A gas-receiver having a volume of 3 cubic feet contains half a pound of oxygen at 70° F. What is the pressure by the gauge?

Here $y = \frac{1}{2}$, $v = 3$, $R = 86.876$, $t = 21° \frac{1}{9}$ C., and $T = 294\frac{1}{9}$.

Therefore,

$$p = \frac{86.876 \times 294\frac{1}{9}}{2 \times 3 \times 144} - 14.697 = 14.876 \text{ lbs. per in.}^2$$

4. A spherical balloon 20 feet in diameter is to be inflated with hydrogen at 60° F., when the barometer stands at 30.2 in., so that gas may not be lost on account of expansion when the

balloon has risen till the barometer stands at 19.6 in., and the temperature falls to 40° F. How many pounds and how many cubic feet of gas are to be run in?

$$\text{Here } v = \frac{4}{3}\pi \times 10^3 = 4188.8 \text{ ft.}^3$$

$$p = \frac{19.6 \times 2116.3}{29.9215} = 1386.8 \text{ lbs. per ft.}^2$$

$$T = 277\tfrac{4}{9} \text{ C.}$$

$$R = 1386.8.$$

$$\therefore y = \frac{p\,v}{R\,T} = 15.092 \text{ lbs.}$$

To determine the number of cubic feet of gas run in, we have

$$v = \frac{y\,R\,T}{p} = 2827.4 \text{ ft.}^3,$$

where

$$p = \frac{30.2 \times 2116.3}{29.9215} = 2136.0.$$

$$T = \frac{5}{9}(60 - 32) + 273 = 288\tfrac{5}{9}.$$

5. "The balloon in which Wellman intends to seek the North Pole has a capacity of 224,244 cubic feet, and weighs, with its car and machinery, 6,600 lbs. What will be its lifting capacity when filled with hydrogen at 10° C. and 760 mm. of the barometer?" (Lissak's "Ordnance and Gunnery," p. 61.)

The balloon, when inflated, will hold at 10° C., 17,458 lbs. of air and 1,209 lbs. of hydrogen. Its lifting capacity will, therefore, be 17,458 − (1,209 + 6,600) = 9,649 lbs.

6. Two pounds of air expand adiabatically from an initial temperature of 60° F., and a pressure of 65.3 lbs. per in.^{2} to a pressure of 50 lbs. per in.^{2} Determine the initial and terminal volumes, the terminal temperature and the external work done.

Here $p_1 = 144 \times 65.3 = 9403.2$; $p = 50 \times 144 = 7200$; $T_1 = 288\frac{5}{9}$ C.; $R = 96.056$; $y = 2$. Take $n = 1.4$

$$\therefore v_1 = \frac{y R T_1}{p_1} = 5.8954 \text{ ft.}^3$$

$$v = v_1 \left(\frac{p_1}{p}\right)^{\frac{5}{7}} = 7.1338 \text{ ft.}^3$$

$$T = T_1 \left(\frac{p}{p_1}\right)^{\frac{2}{7}} = 267.37 \text{ C.} = 481.266 \text{ F.}$$

$$\therefore t = 21.87 \text{ F.}$$

$$W = \frac{y R}{n - 1}(T_1 - T) = 10177 \text{ ft.-lbs.}$$

7. Compute the work of expansion of 2 pounds of air at temperature 100° C., which expands adiabatically until it doubles its volume. Also determine the temperature after expansion and the ratio of the initial and terminal pressures.

Answers:
$$W = 43378 \text{ ft.-lbs.}$$
$$t = 19^\circ.68 \text{ C.}$$
$$p = 0.3789\, p_1.$$

8. A mass of air occupying a volume of 3 ft.^{3} expands adiabatically from an initial temperature of 70° F., and pressure of 85 lbs. per in.^{2}, until external work of 8,000 ft.-lbs. has been done. Compute the terminal volume, pressure, temperature, and weight of air.

Answers:
$$v = 3.768 \text{ ft.}^3$$
$$p = 61.78 \text{ lbs. per in.}^2$$
$$t = 23^\circ.86 \text{ F.}$$
$$y = 1.3 \text{ lbs.}$$

Theoretical Work of an Adiabatic Expansion in the Bore of a Gun.—If, in the first of equations (17), we replace $C_v E T_1$, by its equal $\frac{R T_1}{n - 1}$ it becomes for y pounds of gas

$$W = \frac{y R T_1}{n - 1}\left\{ 1 - \left(\frac{v_1}{v}\right)^{n-1} \right\}$$

This equation gives the work of y pounds of gas at the initial temperature T_1, expanding from the initial volume v_1 to volume v. Suppose the mass of gas to occupy the chamber of a gun with the projectile at its firing seat; and to expand by forcing the projectile along the bore. In this case v_1 will be the volume of the chamber, which is an enlargement of the bore, and is measured by what is called the reduced length of the chamber; that is, by the length of a cylinder whose cross-section is the same as the bore and whose volume is that of the chamber. If u_o is the reduced length required, V_c the volume of the chamber and d the diameter corresponding to the area of cross-section of bore, and which on account of the rifling is slightly greater than the caliber, we evidently have

$$u_o = \frac{4\,V_c}{\pi\,d^2}$$

The variable volume v is the volume of the chamber plus the volume of the bore in rear of the projectile after it has moved any distance u; and is, therefore, measured by $u_o + u$. Therefore the above expression for the work of expansion becomes

$$W = \frac{y\,R\,T_1}{n - 1}\left\{1 - \left(\frac{u_o}{u_o + u}\right)^{n-1}\right\}$$

There is some uncertainty as to the proper value of n for the gases of fired powder. As we have seen, the value of this ratio for perfect gases is approximately 1.4; and it has been generally assumed that at the high temperature of combustion of powder the gases formed may be regarded as possessing all the properties of perfect gases; and therefore most of the earlier writers on interior ballistics employed this value of n in their deductions. But more recent experiments have shown that this value is too great, but have not fixed its true value. The experiments of Noble and Abel with the gases of fired gunpowder, at or near the temperature of combustion, made $n = 1\frac{1}{3}$ nearly;

and this is the value which, for want of a better, we will adopt in what follows. Introducing this value of n into the above expression for the work of expansion; and making $R\,T_1 = f$ and the ratio $u/u_0 = x$, we have

$$W = 3fy \left\{ 1 - \frac{1}{(1 + x)^{\frac{1}{3}}} \right\} \quad . \quad . \quad . \quad (18)$$

The work of expansion in the bore of a gun is expended in many ways, but chiefly in the energy of translation imparted to the projectile. If we assume that the entire work is thus expended, we shall have

$$v^2 = \frac{2g}{w} W = \frac{6gfy}{w} \left\{ 1 - \frac{1}{(1 + x)^{\frac{1}{3}}} \right\} \quad . \quad . \quad . \quad (19)$$

It is evident from (2) that f is the pressure per unit of surface of unit weight of gas at temperature T_1. The ratio x is the number of volumes of expansion of y pounds of gas due to the travel u.

The assumption that the work of expansion is measured by the energy of translation of the projectile does not change the form of the second member of (19); and it is evident that by giving to f a suitable value determined by experiment, the equality expressed in (19) may be strictly true. But in this case f ceases to have the value $R\,T_1$ and becomes simply an experimental coefficient.

In English units (pound and foot), f would be theoretically the pressure in pounds per square foot of one pound of gas at temperature T_1 confined in a volume of one cubic foot.

In metric units (kilogramme and decimetre), f would be defined as above, making the proper change of units.

We may deduce a second approximation to the velocity impressed upon the projectile by the expansion of the gas by taking into account the work performed upon the gun and carriage, as well as upon the projectile. We will suppose the gun mounted upon a free-recoil carriage. Let M be the mass

of the gun and carriage, V their velocity at any period of motion and m the mass of the projectile. The expression for the work of expansion will now be

$$2\,W = m\,v^2 + M\,V^2. \qquad (20)$$

A second equation between the velocities v and V can be deduced by equating the momenta of the system projected upon the axis of the gun. We thus obtain

$$m\,v = M\,V \qquad (21)$$

Eliminating V from (20) and (21) there results

$$v^2 = \frac{2\,W}{m\left(1 + \frac{m}{M}\right)} \qquad (22)$$

This expression for v^2 is the same as that given by (19) with the exception of the small fraction m/M which can be safely neglected in comparison with unity. Similarly it may be shown that the work expended upon the projectile in giving it rotation about its axis is small in comparison with the work of translation.

Noble and Abel's Researches on Fired Gunpowder.—Noble and Abel's experiments on the explosion of gunpowder in close vessels were given to the world in two memoirs which were read before the Royal Society in 1874 and 1879, respectively. These experiments have an important bearing upon the subject of interior ballistics, since they furnish the most reliable values we possess of the temperature of combustion of fired gunpowder, the mean specific heat of the products of combustion (solid as well as gaseous), the ratio of solid to gaseous products, and, lastly, what is known as the *force of the powder*,—all of which are important factors in computing the work done by the gases of a charge of gunpowder exploded in the chamber of a gun.

The vessels in which the explosions were produced were of two sizes, the smaller one for moderate charges and for experi-

ments connected with the measurement or analysis of the gases, while in the larger one Captain Noble states that he has succeeded in absolutely retaining the products of combustion of a charge of 23 pounds of gunpowder.* These vessels consisted of a steel barrel open at both ends, the two open ends being closed by carefully fitted screw plugs (firing plug and crusher plug), furnished with gas checks to prevent any escape of gas past the screw. In the firing plug was a conical hole closed from within by a steel cone which was ground into its place with great exactness, and which, when the cylinder was prepared for firing, was covered with very fine tissue paper to give it electrical insulation from the rest of the apparatus. The two wires from a Leclanché battery were attached, the one to the insulated cone and the other to the firing plug, and were connected within the powder chamber by a fine platinum wire passing through a glass tube filled with mealed powder. This platinum wire became heated when the electric current passed through it, and the charge was thus fired. At the opposite end of the cylinder from the firing plug was another plug fitted with a crusher gauge for determining the pressure of the gases. The vessel was also provided with an arrangement for collecting the gases after an explosion for analysis, measurement of quantity, or for other purposes.

Results of the Experiments.—It was found that about 57 per cent. by weight of the products of combustion were non-gaseous, consisting principally of potassium carbonate, potassium sulphate, and potassium sulphide, the first named greatly preponderating. The remaining 43 per cent. were permanent gases, principally CO_2, CO and N. These gases, when brought to a temperature of 0° C., and under the normal atmospheric pressure of 760 millimetres, occupied about 280 times the volume of the unexploded powder.

* Lecture on Internal Ballistics, by Captain Noble, London, 1892, p. 12.

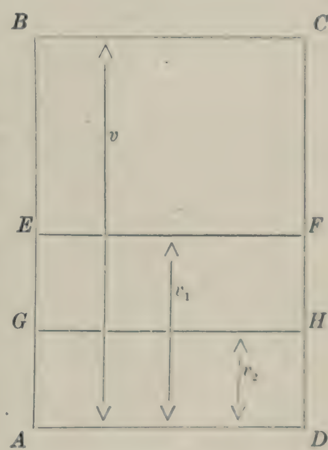
Pressure in Close Vessels, Deduced from Theoretical Considerations.—The expression for the pressure of the gases developed by the combustion of gunpowder in a close vessel is deduced upon the following suppositions:

1st. That a portion of the products of combustion is in a liquid state.

2d. That the pressure due to the permanent gases can only be calculated by deducting the volume of the liquid products from the volume of the vessel.

Upon these hypotheses the expression for the pressure may be deduced as follows:

Let $A\ B\ C\ D$ be a section of a close vessel of volume v in which a given charge of powder is exploded.



Let $A\ E\ F\ D$ represent the space (v_1) occupied by the charge, and $A\ G\ H\ D$ the space (v_2) occupied by the non-gaseous products. Let Δ_1 be the so-called density of the products of combustion,—that is $\Delta_1 = \frac{v_1}{v}$; and α the ratio of the non-gaseous products to the volume of the charge, or $\alpha = \frac{v_2}{v_1} = \frac{v_2}{\Delta_1 v}$. The gases after explosion will occupy the space $v - v_2 = v - \alpha\ \Delta_1 v = v\,(1 - \alpha\ \Delta_1)$. Let p_1 be the pressure that would be developed if the volume of the vessel were $A\ E\ F\ D$ (or v_1). In this case the density of the products of combustion (Δ_1) (the charge remaining the same) would be unity; and the space occupied by the gases would be $v_1 - v_2 = v_1\,(1 - \alpha) = \Delta_1 v\,(1 - \alpha)$. Now if p is the pressure when the volume of the vessel is v, we have by Mariotte's law (assuming that the temperature is the same for all densities of the products of combustion),

$$p = p_1 \frac{\Delta_1 v(1-\alpha)}{v(1-\alpha\Delta_1)} = (1-\alpha)\, p_1 \frac{\Delta_1}{1-\alpha\Delta_1};$$

or, making

$$f = (1-\alpha)\, p_1,$$

we have

$$p = f \frac{\Delta_1}{1-\alpha\Delta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The factor f is called the *force of the powder.*

Value of the Ratio α.—Let p_2 and p_3 be the pressures in the same vessel produced by two different charges, and Δ_2 and Δ_3 the corresponding densities of the products of combustion. Then from equation (23) (assuming f to be the same for all values of Δ_1),

$$p_2 = f \frac{\Delta_2}{1-\alpha\Delta_2},$$

and

$$p_3 = f \frac{\Delta_3}{1-\alpha\Delta_3};$$

whence by division,

$$\frac{1-\alpha\Delta_3}{1-\alpha\Delta_2} = \frac{p_2\,\Delta_3}{p_3\,\Delta_2}.$$

Therefore

$$\alpha = \frac{1}{\Delta_2\,\Delta_3} \left\{ \frac{p_3\Delta_2 - p_2\Delta_3}{p_3 - p_2} \right\},$$

by means of which the mean value of α can be determined when a sufficient number of pressures, corresponding to different values of Δ_1, have been found by experiment. The value of α finally adopted by Noble and Abel is 0.57.

Determination of the Force of the Powder.—To determine f we have from equation (23),

$$f = p \frac{1 - .57\,\Delta_1}{\Delta_1};$$

from which f may be found by means of a single measured pressure corresponding to a given density of the products of

combustion. When $\Delta_1 = 1$, that is, when the vessel is completely filled by the charge, p was found to be 43 tons per square inch, and therefore $f = 43\ (1 - .57) = 18.49$ tons or 41417.6 pounds per square inch. Therefore Noble and Abel's formula for the pressure in a close vessel is, for different densities of the products of combustion,

$$\left.\begin{aligned} p &= 18.49 \frac{\Delta_1}{1 - .57\ \Delta_1} \text{ tons per sq. in.} \\ &= 41417.6 \frac{\Delta_1}{1 - .57\ \Delta_1} \text{ lbs. per sq. in.} \end{aligned}\right\} \quad . \quad . \quad (24)$$

To transform this equation so that it shall express the pressure in kilos per dm.^{2} we may employ a simple rule which, as it is of frequent use, is here inserted for convenience:

RULE:—To reduce a pressure expressed in tons per square inch to the same pressure expressed in kilos per dm.^{2}, add to the logarithm of the former the constant logarithm 4.1972544 and the sum is the logarithm of the pressure required.

If the pressure to be reduced is in pounds per in.^{2} then the constant logarithm to be added is 0.8470064.

Applying this rule the expression for the pressure of the products of combustion of a charge of gunpowder fired in a close vessel is found to be

$$p = 291200 \frac{\Delta_1}{1 - .57\,\Delta_1}$$

$$\therefore f = 291200 \text{ kilos per dm.}^2$$

It will be seen from the definition given to Δ_1 that it is the *density of loading* as defined in Chapter III when the gravimetric density of the powder is unity,—that is, when a kilo of the powder fills a volume of a dm.^{3}; or, what is the same thing, when a pound occupies a volume of 27.68 cubic inches; and in this case, when Δ_1 is unity the charge just fills the receptacle. Noble and Abel were careful to keep the gravimetric density of the powder they experimented with as near unity as possible.

Interpretation of f.—It will be seen from Equation (23) that the quantity designated by f is the pressure of the gases when

$$\frac{\Delta_1 v}{v(1 - \alpha \Delta_1)} = 1,$$

that is, when the space occupied by the gases is equal to the volume of the charge, which requires that the vessel should have $1 + \alpha$ units of volume. Thus if the kilogramme and litre are the units of weight and volume, respectively, the volume of the vessel must be 1.57 litres in order that the gases may occupy a volume of one litre, and have a tension equal to f. From this f may be defined to be the pressure of the gases of unit weight of powder occupying unit volume at the temperature of combustion T_1.

If ϵ is the weight of gas furnished by the combustion of unit weight of powder we have from Equation (2),

$$p_1 v_1 = \epsilon R T_1;$$

and if v_1 is the unit of volume, there results

$$p_1 = f = \epsilon R T_1 \quad . \quad . \quad . \quad . \quad . \quad (25)$$

If the pound is the unit of weight the unit of volume is 27.68 cubic inches. In this case the definition of f requires that the volume of the vessel should be $1.57 \times 27.68 = 43.459$ cubic inches.

The value of ϵ, according to Noble and Abel, is 0.43; and therefore the pressure of unit weight of the gases of fired gunpowder at temperature T_1 is

$$\frac{f}{0.43}.$$

From this it follows that the pressure of one pound of the gases of fired gunpowder at temperature of combustion, confined in a volume of 27.68 cubic inches, is

$$\frac{41417.6}{0.43} = 96320 \text{ lbs. per square inch.}$$

Also, the pressure of one pound of the gases of the paragraph immediately preceding, confined in a volume of one cubic foot, is, in pounds per square foot,

$$\frac{96320 \times 27.68}{12} = 222180 \text{ lbs.}$$

If the gravimetric density of the powder be unity, and y and v be taken in pounds and cubic inches, respectively, then Equation (23) becomes

$$p = f \frac{27.68\, y}{v - 27.68\, \alpha\, y} \quad . \quad . \quad . \quad . \quad (26)$$

Solving with reference to y and to v gives

$$y = \frac{p\, v}{27.68\, (\alpha\, p + f)} \quad . \quad . \quad . \quad . \quad (27)$$

and

$$v = \frac{27.68\, y\, (\alpha\, p + f)}{p} \quad . \quad . \quad . \quad . \quad (28)$$

These equations are useful in questions involving the bursting of shells, etc.

Theoretical Determination of the Temperature of Explosion of Gunpowder.—Having determined the value of f from the experiments, we can deduce the temperature of explosion by means of the formula

$$T_1 = \frac{273}{\epsilon\, p_o\, v_o} f$$

According to Noble and Abel's experiments, if the gravimetric density of the powder is such that a kilogramme occupies one litre, the gases furnished by its combustion will fill a volume of 280 litres at 0° C. under the normal atmospheric pressure of 103.33 kgs. per square decimetre. We therefore have

$$v_o = \frac{280}{y\, \epsilon}$$

and

$$p_o = 103.33$$

whence

$$T_1 = \frac{273 \times 291200}{103.33 \times 280} = 2748^\circ \text{ C.}$$

This is the absolute temperature of combustion of gunpowder according to Noble and Abel's latest deductions from their experiments. Subtracting 273° from this temperature we have temperature of explosion = 2475° C. (4487° F.).

Mean Specific Heat of the Products of Combustion.—From equation (8), we have when $W = 0$, that is, when no external work is performed,

$$Q = C_v (T_1 - 273)$$

in which Q is the heat of combustion; that is, the quantity of heat that unit of weight of the explosive substance evolves, under constant volume, when the final temperature of the products of combustion is 0° C. From this equation we find

$$C_v = \frac{Q}{T_1 - 273}$$

The heat of combustion was determined by Noble and Abel in the following manner:

"A charge of powder was weighed and placed in one of the smaller cylinders, which was kept for some hours in a room of very uniform temperature. When the apparatus was throughout of the same temperature, the thermometer was read, the cylinder closed, and the charge exploded.

"Immediately after explosion the cylinder was placed in a calorimeter containing a given weight of water at a measured temperature, the vessel being carefully protected from radiation, and its calorific value in water having been previously determined.

"The uniform transmission of heat through the entire volume of water was maintained by agitation of the liquid, and the thermometer was read every five minutes until the maximum

was reached. The observations were then continued for an equal time to determine the loss of heat in the calorimeter due to radiation, etc.; the amount so determined was added to the maximum temperature."

In this way the heat of combustion of R. L. G. and F. G. powders was found to be 705 heat-units; that is, the combustion of a unit weight of the powder liberated sufficient heat to raise the temperature of 705 unit-weights of water 1° C. We therefore have

$$C_v = \frac{705}{2475} = 0.285$$

This result is accepted by Noble and Abel, and also by Sarrau, as a very close approximation to the mean specific heat of the entire products of combustion. If we assume that the mean specific heat of gunpowder of different compositions is constant, we can compute the temperatures of combustion when the heat of combustion has been determined by the calorimeter, by the formula

$$T = \frac{Q}{0.285}$$

in which T is given by the centigrade scale.

Pressure in the Bores of Guns Derived from Theoretical Considerations.—"At an early stage in our researches, when we found, contrary to our expectation, that the elastic pressure deduced from experiments in close vessels did not differ greatly (where the powder might be considered entirely consumed, or nearly so) from those deduced from experiments in the bores of guns themselves, we came to the conclusion that this departure from our expectation was probably due to the heat stored up in the liquid residue. In fact, instead of the expansion of the permanent gases taking place without addition of heat, the residue, in the finely divided state in which it must be on the ignition of the charge, may be considered a source of heat of the most per-

fect character, and available for compensating the cooling effect due to the expansion of the gases on production of work.

"The question, then, that we now have to consider is—What will be the conditions of expansion of the permanent gases when dilating in the bore of a gun and drawing heat, during their expansion, from the non-gaseous portions in a very finely divided state ?" *

Let $c_{\prime}$ be the specific heat of the non-gaseous portion of the charge, which we can assume, without material error, to be constant. We shall then have $c_{\prime}\, d\, T$ for the elementary quantity of heat yielded to the gases per unit of weight of liquid residue. If there are $w_{\prime}$ units of weight of liquid residue it will yield to the gases $w_1\, c_{\prime}\, d\, T$ units of heat; and if there are w_2 units of weight of gas we shall have in heat-units,

$$d\, q = -\frac{w_1}{w_2}\, c_{\prime}\, d\, T = -\beta\, c_{\prime}\, d\, T,$$

in which

$$\beta = \frac{w_1}{w_2};$$

that is, β is the ratio between the weights of the non-gaseous and gaseous portions of the charge. The negative sign is given to the second member because T decreases while q increases.

Substituting the above value of $d\, q$ in Equation (7), it becomes

$$-\,(C_v + \beta\, c_{\prime})\, d\, T = \frac{C_p - C_v}{R}\, p\, d\, v\; . \quad . \quad . \quad . \quad (29)$$

and this combined with Equation (6), gives, by a slight reduction,

$$-\,(\beta\, c_{\prime} + C_p)\,\frac{d\, v}{v} = (\beta c_{\prime} + C_v)\,\frac{d\, p}{p}\quad . \quad . \quad . \quad (30)$$

Since C_p, C_v, c_1 and β are, by hypothesis, constant during the expansion, the integration of Equation (30) between the limits v_2

* Noble and Abel, Researches, etc., page 98.

and v_3—the former being the initial volume occupied by the permanent gases and the latter their volume after the projectile has been displaced by a distance u, gives

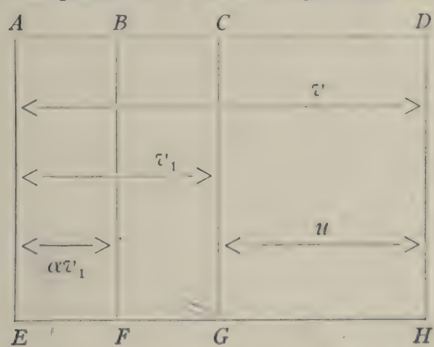
$$p = p_1\left(\frac{v'_2}{v'_3}\right)^r \quad . \quad . \quad . \quad . \quad . \quad (31)$$

in which

$$r = \frac{\beta c_, + C_p}{\beta c_, + C_v}$$

Equation (31), it will be seen, becomes identical with Equation (15), when $\beta = 0$; that is, when there is no liquid residue.

To introduce v_1 and v, that is the volumes occupied by the charge and the entire volume in the rear of the projectile, into Equation (30) in place of v_2 and v_3, proceed as follows: Let



$A\ C\ E\ G$ represent the chamber of the gun, which we will suppose filled with powder without compression, and further that one pound of the powder fills a space of 27.68 cubic inches. The gravimetric density and density of loading are each unity; and if v_1 is the volume of the chamber, it follows that

$$v_1 = 27.68\ \bar{\omega}.$$

$\bar{\omega}$ being the weight of charge.

Suppose the powder to be entirely consumed before the projectile moves any perceptible distance; and that the non-gaseous products fill the space $A\ B\ E\ F$, whose volume is $\alpha\ v_1$. The gases,

therefore, which by their expansion give motion to the projectile will occupy the space $B\,C\,F\,G$ before perceptible motion begins. The volume of the space $B\,C\,F\,G$ is evidently $v_2 = v_1 - \alpha\, v_1 = v_1\,(1 - \alpha)$. Let $D\,H$ be the base of the projectile after it has moved a distance u; and, designating the volume $A\ D\ E\ H$ by v, we evidently have $v_3 = v - \alpha\, v_1$. Substituting these values of v_2 and v_3 in Equation (31) gives

$$p = p_1 \left(\frac{v_1\,(1 - \alpha)}{v - \alpha\, v_1}\right)^r \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

In this equation p_1 is the pressure produced by the combustion of a charge of powder in a close vessel when the density of loading is unity. The values of the constants are given by Noble and Abel as follows: *

$$\begin{aligned} p_1 &= 43 \text{ tons per square inch} \\ \alpha &= 0.57 \\ \beta &= 1.2957 \\ C_p &= 0.2324 \\ C_v &= 0.1762 \\ c_{\prime} &= 0.45 \\ v_1 &= 27.68\,\tilde{\omega} \end{aligned}$$

from which we find $r = 1.074$. Substituting these values in the expression for p it becomes

$$p = 43 \left(\frac{0.43\, v_1}{v - 0.57\, v_1}\right)^{1.074} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

which gives the pressure in tons per square inch.

If, as in a close vessel, we let

$$\Delta_1 = \frac{v_1}{v},$$

then

$$p = (0.43)^{1.074} \times 43 \left\{\frac{\Delta_1}{1 - .57\,\Delta_1}\right\}^{1.074}$$

* Researches, etc., page 167.

$$= 17.375 \left\{ \frac{\Delta_1}{1 - .57\,\Delta_1} \right\}^{1.074} \quad . \quad . \quad . \quad (34)$$

REMARKS:—The value of $\beta = 1.2957$, adopted by Noble and Abel, gives for unit weight, $w_1 = 0.5644$ and $w_2 = 0.4356$, while the values of these quantities adopted in our equations are 0.57 and 0.43, respectively. These last-named values would make $\beta = 1.3256$.

Noble and Abel's values of the specific heats of the permanent gases of combustion, namely, $C_p = 0.2324$ and $C_v = 0.1762$, make $n = 1.32$; while for perfect gases, as has been shown, $n = 1.4$ very nearly.

Table of Pressures.—In the following table of pressures the third column gives the pressures in the bore of a gun corresponding to the values of Δ_1 in the first column. They were computed by Equation (34) upon the assumption that the permanent gases in expanding, and thereby doing work, borrow heat from the non-gaseous residue; and also that the combustion is complete before the projectile has moved perceptibly; and finally that there is no conduction of heat to the walls of the gun. The tensions in the fifth column were computed by Equation (24) and agree with Noble and Abel's experiments.*

Temperatures of Products of Combustion in Bores of Guns.—The temperature in the bore of a gun during the expansion of the products of combustion, may be determined from Equation (29), which replacing R by its value from Equation (1), becomes

$$\frac{dT}{T} = -\frac{C_p - C_v}{\beta c_{\prime} + C_v}\frac{dv}{v};$$

whence integrating between the same limits as before, and observing that

$$\frac{C_p - C_v}{\beta c_{\prime} + C_v} = r - 1,$$

* For table of pressures see page 46.

TABLE OF PRESSURES.

Mean density of products of combustion. Δ_1	Corresponding expansions. $\frac{1}{\Delta_1}$	Tensions calculated by Equation (33).		Tensions in close cylinders, or where gases expand without doing work.	
		Tons per square inch.	Differences.	Tons per square inch.	Differences.
1.00	1.000	43.00	5.01	43.00	4.69
.95	1.053	37.99	4.40	38.31	4.14
.90	1.111	33.59	3.88	34.17	3.68
.85	1.176	29.72	3.44	30.49	3.30
.80	1.250	26.28	3.06	27.19	2.97
.75	1.333	23.22	2.76	24.22	2.68
.70	1.429	20.46	2.48	21.54	2.45
.65	1.539	17.98	2.25	19.09	2.23
.60	1.667	15.73	2.04	16.86	2.05
.55	1.818	13.69	1.86	14.81	1.88
.50	2.000	11.83	1.70	12.93	1.74
.45	2.222	10.13	1.56	11.19	1.61
.40	2.500	8.57	1.43	9.58	1.50
.35	2.857	7.14	1.31	8.08	1.39
.30	3.333	5.83	1.21	6.69	1.30
.25	4.000	4.62	1.11	5.39	1.22
.20	5.000	3.51	1.02	4.17	1.14
.15	6.667	2.49	.93	3.03	1.07
.10	10.000	1.56	.84	1.96	1.01
.05	20.000	.72	....	.95	....

we have

$$T = T_1 \left(\frac{v_2}{v_3}\right)^{r - 1}.$$

Replacing v_2 by $v_1\ (1 - \alpha)$, and v_3 by $v - \alpha\, v_1$, for reasons already given, we have for the absolute temperature of the gases during expansion, the equation

$$T = T_1 \left(\frac{v_1\,(1 - \alpha)}{v - \alpha\, v_1}\right)^{r - 1}.$$

Introducing the density of the products of combustion (Δ_1), and the numerical values of α and r into this last equation, it becomes

$$T = (.43)^{.074} \times T_1 \left\{ \frac{\Delta_1}{1 - .57\,\Delta_1} \right\}^{.074}$$

$$= 0.93946\, T_1 \left\{ \frac{\Delta_1}{1 - .57\,\Delta_1} \right\}^{.074} \quad . \quad . \quad . \quad (35)$$

The value of T for any given density of the products of combustion (represented by Δ_1) will depend upon their initial temperature (or absolute temperature of combustion), T_1. Its theoretical value, based upon Noble and Abel's latest deductions from their experiments, as published in their second memoir, has already been found to be 2748° C. But there are very great difficulties in the way of verifying by experiment the theoretical value of T_1, and Captain Noble in his Greenock lecture (February 12th, 1892) takes the absolute temperature of combustion at 2505° C., as deduced in their first memoir. Therefore making

$$T_1 = 2505° \text{ C.},$$

the expression for T becomes

$$T = 2353.3 \left\{ \frac{\Delta_1}{1 - .57\,\Delta_1} \right\}^{.074} \quad . \quad . \quad . \quad (36)$$

The temperatures in degrees Centigrade and Fahrenheit, calculated from Equation (36), are given in the following table. "It is hardly necessary to point out that the values given in this table are only strictly accurate when the charge is ignited before the projectile is sensibly moved; but in practice the correction due to this cause will not be great." *

Theoretical Work Effected by Gunpowder.—The theoretical work which a charge of gunpowder is capable of effecting during the expansion of its volume from v_1 to any volume v is expressed by the definite integral

$$W = \int_{v_1}^{v} p\, d\, v;$$

* For table of temperatures see next page.

TABLE OF TEMPERATURES.

Mean density of products of combustion. Δ_1	Number of volumes of expansion. $\frac{1}{\Delta_1}$	Temperatures.	
		Centigrade.	Fahrenheit.
1.00	1.0000	2231	4048
.95	1.0526	2210	4010
.90	1.1111	2189	3972
.85	1.1765	2168	3934
.80	1.2500	2147	3897
.75	1.3333	2126	3859
.70	1.4286	2106	3823
.65	1.5385	2085	3785
.60	1.6667	2063	3745
.55	1.8182	2041	3706
.50	2.0000	2018	3664
.45	2.2222	1994	3621
.40	2.5000	1968	3574
.35	2.8571	1940	3524
.30	3.3333	1909	3468
.25	4.0000	1874	3405
.20	5.0000	1834	3333
.15	6.6667	1785	3245
.10	10.0000	1719	3126
.05	20.0000	1615	2939
.00	∞	0	0

or, substituting for p its value from (32),

$$W = \int_{v_1}^{v} p_1 \left(\frac{v_1(1-\alpha)}{v-\alpha v_1}\right)^r d\,v,$$

$$= p_1\, v_1^r (1-\alpha)^r \int_{v_1}^{v} \frac{d\,v}{(v-\alpha\, v_1)^r};$$

whence, integrating, we have

$$W = \frac{p_1 v_1^r (1-\alpha)^r}{r-1} \left\{ \frac{1}{[v_1(1-\alpha)]^{r-1}} - \frac{1}{(v-\alpha\, v_1)^{r-1}} \right\}.$$

Multiplying and dividing the second member by $[v_1\,(1-\alpha)]^{r-1}$, we have

$$W = \frac{p_1 v_1(1-\alpha)}{(r-1)} \left\{ 1 - \left(\frac{v_1(1-\alpha)}{v-\alpha\, v_1}\right)^{r-1} \right\}.$$

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If, in this last equation, p_1 be expressed in kilogrammes per square decimetre, and v_1 be made unity (one litre), the work will be expressed in decimetre-kilogrammes per kilogramme of powder burned. To express the work in foot-tons per pound of powder burned, we must make $v_1 = 27.68$ cubic inches; and then, since p_1 is given in tons per square inch, divide the result by 12, the number of inches in a foot. Making these substitutions and replacing α and r by their values already given, we have, in foot-tons,

$$W = 576.369 \left\{ 1 - \left(\frac{.43\, v_1}{v - .57\, v_1} \right)^{r-1} \right\};$$

or, in terms of Δ_1,

$$W = 576.369 \left\{ 1 - 0.93946 \left(\frac{\Delta_1}{1 - .57\, \Delta_1} \right)^{.074} \right\} \quad . \quad (37)$$

Substituting in Equation (37) from Equation (35) we have

$$W = \frac{576.369}{T_1} (T_1 - T)$$

or, since, according to Noble and Abel,

$$T_1 = 2505°$$

we have

$$W = 0.23008\, (T_1 - T) \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

which gives the work in terms of the loss of temperature of the products of combustion.

Table III gives the work of expansion of the gases of one pound of gunpowder of the normal type and free from moisture, computed by Equation (37). By means of the work given in this table, and by the use of a proper factor of effect determined by experiment, Noble and Abel consider that the actual work of a given charge of powder upon a projectile may be computed with considerable accuracy. Their method of using this table will be clearly seen by the following extract:

"If we wish to know the maximum work of a given charge,

fired in a gun with such capacity of bore that the charge suffered five expansions ($\Delta_1 = 0.2$) during the motion of the projectile in the gun, the density of loading being unity, the table shows us that for every pound in the charge, an energy of 91.4 foot-tons will as a maximum be generated.

"If the factor of effect for the powder and gun be known, the above values, multiplied by that factor, will give the energy per pound that may be expected to be realized in the projectile.

"But it rarely happens, especially with the very large charges used in the most recent guns that densities of loading so high as unity are employed; and in such cases, from the total energy realizable must be deducted the energy which the powder would have generated, had it expanded from a density of unity to that actually occupied by the charge. Thus in the example above given, if we suppose the charge instead of a density of loading of unity to have a density of 0.8, we see from Table 3, that from the 91.4 foot-tons above given, there must be subtracted 19.23 foot-tons; leaving 72.17 foot-tons as the maximum energy realizable under the given conditions, per pound of the charge." *

To apply these principles practically for *muzzle velocities*, let, as before,

v_1 be the volume occupied by the charge, in cubic inches.
v the total volume of bore and chamber, in cubic inches.
V_b the volume of the bore.
V_c the volume of the chamber, in cubic inches.

Then

$$v = V_b + V_c \,;$$

and, if the gravimetric density of the powder be unity,

$$v_1 = 27.68\,\tilde{\omega},$$

where $\tilde{\omega}$ is the weight of the charge in pounds. Therefore the

* Noble and Abel, Researches, page 176.

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number of volumes of expansion of the products of combustion will be, at the muzzle,

$$\frac{v}{v_1} = \frac{1}{\Delta_1} = \frac{V_b}{27.68\,\bar{\omega}} + \frac{V_c}{v_1};$$

which may be written, if the gravimetric density of the powder be unity,

$$\frac{1}{\Delta_1} = 0.0361263\frac{V_b}{\bar{\omega}} + \frac{1}{\Delta}$$

in which Δ is the density of loading as defined in Chapter III.

If the gravimetric density of the powder be not unity, let v_2 be the volume in cubic inches of one pound of powder not pressed together except by its own weight; and let

$$\frac{27.68}{v_2} = m;$$

then we have in all cases,

$$\frac{1}{\Delta_1} = m\left\{0.0361263\frac{V_b}{\bar{\omega}} + \frac{1}{\Delta}\right\},$$

in which $\frac{1}{\Delta_1}$ is the number of volumes of expansion of the products of combustion.

Let W_2 be the work taken from Noble and Abel's table (Table III) of the gases of one pound of powder for a given value of $\frac{1}{\Delta_1}$, and W_1 the work due to the expansion $\frac{m}{\Delta}$. Also, let F be the factor of effect. Then if we assume that the work of expansion is all expressed in the energy of translation of the projectile, we shall have approximately,

$$\frac{w\,v^2}{4480\,g} = F\,W\,\bar{\omega} \qquad . \quad . \quad . \quad . \quad . \quad (39)$$

in which w is the weight of the projectile and

$$W = W_2 - W_1$$

From (39) the muzzle velocity v may be computed when the

factor of effect is known; or, we may determine the factor of effect when the muzzle velocity has been measured by a chronograph. These two equations reduced to practical forms are the following:

$$v = 379.57 \sqrt{F W \frac{\bar{\omega}}{w}} \quad . \quad . \quad . \quad . \quad (40)$$

and

$$F = 0.000006941 \frac{v^2 w}{W \bar{\omega}} \quad . \quad . \quad . \quad . \quad (41)$$

As an illustrative application of these formulas to interior ballistics take the following data from Noble and Abel's second memoir, relative to the English 8-inch gun: It was found by firing a charge of 70 pounds of a certain brand of pebble powder, with a projectile weighing 180 pounds, that a muzzle velocity of 1694 foot-seconds was obtained. What was the factor of effect (F) pertaining to this gun and brand of powder? For this particular gun and charge we have $\bar{\omega} = 70$ pounds, $w = 180$ pounds, $\Delta_1 = 0.1634$, $\Delta = 0.605$ and $m = 1$. In Noble and Abel's table of work (Table III) the first column gives values of $\frac{1}{\Delta_1}$, increasing by a common difference, while the second column contains the corresponding values of Δ_1. By a simple interpolation we find for the values of Δ_1 and Δ given above, $W_2 = 99.4$ and $W_1 = 37.6$; whence $W = 61.8$ foot-tons. Substituting these values in Equation (41) we have

$$F = 0.000006941 \frac{180 \times (1694)^2}{70 \times 61.08} = 0.8287.$$

That is, the actual work realized, as expressed and measured by the projectile's energy of translation, as it emerges from the bore, is nearly 83 per cent. of the theoretical maximum work which the powder gases are capable of performing, leaving but 17 per cent. for the other work done by the gases, namely, the work expended upon the charge, the gun and carriage, and in

giving rotation to the projectile; the work expended in overcoming passive resistances, such as forcing the rotating band into the groove, the subsequent friction as the projectile moves along the bore, and the resistance of the air in front of the projectile; and lastly, the heat communicated to the walls of the gun. It is very difficult to evaluate these non-useful energies, but it is probable that they do not consume more than 17 per cent. of the maximum work of the gases. Longridge finds by an elaborate calculation that this lost work in a 10-inch B. L. Woolwich gun amounts to 30 per cent. of the maximum work; * but it is believed that he has greatly overestimated the work required to give motion to the products of combustion. Colonel Pashkievitsch makes the lost work rather less than 17 per cent. of that expressed in the energy of translation of the projectile.†

To test the correctness of Equation (40) for determining muzzle velocities we will apply it to the same gun by means of which the factor of effect was determined, increasing the charge from 70 to 90 pounds, and again to 100 pounds, and compare the computed velocities with those measured with a chronograph. For a charge of 90 pounds of powder we have $\Delta_1 = 0.210$ and $\Delta = 0.780$; whence $W_2 = 89.3$, $W_1 = 20.86$, and $W = 68.44$

$$\therefore v = 379.57\sqrt{\frac{.8287 \times 90 \times 68.44}{180}} = 2021 \text{ foot-sec.}$$

The measured velocity with this charge was 2027 foot-seconds. In a similar way we find by the formula that for a charge of 100 pounds $v = 2174$ foot-seconds, while the measured velocity was 2182 foot-seconds. The differences between the computed and observed velocities in these examples are about one-third of one

* "Internal Ballistics." By Atkinson Longridge. London, 1889. Chapter V.

† "Interior Ballistics." By Colonel Pashkievitsch. Translated from the Russian by Captain Tasker H. Bliss, U. S. Army. Washington, 1892.

per cent., and are well within the limits of probable error in measuring them.

The factor of effect increases with the caliber of the gun, as is shown by experiment. Thus with the English 10-inch gun fired with charges of 130 and 140 pounds of the pebble powder we have been considering, the factor of effect is 0.855; while with the 11-inch gun, and charge of 235 pounds, the factor of effect is 0.89.

CHAPTER III

COMBUSTION UNDER CONSTANT PRESSURE

Combustion of a Grain of Powder Under Constant Atmospheric Pressure.—In what follows it is assumed that the powder grain is of some regular geometrical form to which the elementary rules of mensuration can be applied. It will also be assumed as the result of observation, that the combustion of the grain takes place simultaneously on all sides and that, under the constant pressure of the atmosphere, parallel layers of equal thickness are burned away in equal successive intervals of time—that is, that the velocity of combustion under constant pressure is uniform.

The form and dimension of each grain of powder constituting the charge are of the utmost importance, as upon them depends the proper distribution of the mean effective pressure within the bore. If the initial surface of combustion of the charge be large and the web thickness of the grains small, then the maximum pressure will be excessive and the muzzle velocity inadequate. On the other hand, if the web thickness be too great the chase pressure may prove destructive to the gun. More than one of our heavy guns it is believed have been wrecked during the past ten years simply from excessive web thickness.

Many forms of grain have been adopted by different manufacturers in this and foreign countries, but they may all be divided into two general groups, *viz.*: those burning with a continuously decreasing surface, and those in which the surface of combustion may increase (or decrease) to a certain stage, the grain then breaking up into other forms entirely dissimilar to the original and which are then consumed with a rapidly decreasing surface. To the first group belong spherical, cubical, ribbon-shaped, and indeed all solid grains of whatever form,

and cylindrical grains with an axial perforation. To the latter group belong pierced prismatic and the so-called multiperforated grains employed by both our army and navy.

Notation.—Let

l = thickness of layer burned in time t.

l_o = one-half the least dimension of the grain. Since combustion takes place on all sides of a grain at once, it may be assumed that when $l = l_o$ all grains of the first group are totally consumed. This, of course, is not the case with m. p. grains.

S_o = the total initial surface of combustion of the grain.

S = surface of combustion at time t, corresponding to l.

S' = the total burning surface when $l = l_o$; that is, when the grain, as a grain, is about to disappear. This surface may be called the vanishing surface of combustion.

V_o = the initial volume of a grain.

V = volume of grain burned at time t. That is, the volume comprised between the surfaces S_o and S.

k = fraction of grain burned in time t. That is, $k = \dfrac{V}{V_o}$.

The general expression for the burning surface of a grain of powder moulded into any one of the simple geometrical forms adopted by powder manufacturers may take the form,

$$S = S_o + a\,l + b\,l^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where l is the thickness of layer burned from instant of ignition. At that instant l is zero and S the initial surface of combustion S_o. In the course of burning when l is about to become l_o, S is about to become S'. Therefore

$$S' = S_o + a\,l_o + b l_o^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In these two equations a and b are constants for the same form of grain, whose values will be deduced later.

The general expression for the volume consumed while a thickness l is burned away, is

$$V = \int_o^l S\,d\,l\,;$$

whence substituting for S its general value from (1) and integrating,

$$V = S_o\, l + \frac{a}{2} l^2 + \frac{b}{3} l^3 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The initial volume V_o is evidently what V becomes when the grain is completely consumed, that is, when $l = l_o$. Therefore

$$V_o = S_o\, l_o + \frac{a}{2} l_o^2 + \frac{b}{3} l_o^3 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This, of course, gives the entire original volume only for those grains which are completely consumed when $l = l_o$, or, in other words, when the web thickness is burned. It need hardly be said that it does not apply to m. p. grains. In this latter case, it gives the original volume minus the "slivers," so called.

If, in (4), we substitute for S_o its value from (2), namely,

$$S_o = S' - a\, l_o - b\, l_o^2$$

it becomes

$$V_o = S'\, l_o - \frac{a}{2} l_o^2 - \frac{2\, b}{3} l_o^3 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From (4) and (5) we readily find

$$a = -\frac{2}{l_o}(2\, S_o + S') + \frac{6\, V_o}{l_o^2} \quad . \quad . \quad . \quad (6)$$

and

$$b = \frac{3}{l_o^2}(S_o + S') - \frac{6\, V_o}{l_o^3} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

These equations give a and b when S_o, S' and V_o can be computed by the rules of mensuration. It will be observed that a is a linear quantity while b is of zero order of magnitude. These properties afford tests, as far as they go, as to whether the work of deducing a and b in any particular case has been correctly performed.

Fraction of Grain Burned for any Value of l.—We have by definition

$$k = \frac{V}{V_o} = \frac{S_o l + \frac{a}{2} l^2 + \frac{b}{3} l^3}{V_o}.$$

This may be transformed into

$$k = \frac{S_o l_o}{V_o} \cdot \frac{l}{l_o} \left\{ 1 + \frac{a l_o}{2 S_o} \cdot \frac{l}{l_o} + \frac{b l_o^2}{3 S_o} \cdot \frac{l^2}{l_o^2} \right\}.$$

Put for convenience,

$$\frac{S_o l_o}{V_o} = \alpha; \quad \frac{a l_o}{2 S_o} = \lambda; \quad \frac{b l_o^2}{3 S_o} = \mu . \quad . \quad . \quad . \quad (8)$$

Then

$$k = \alpha \frac{l}{l_o} \left\{ 1 + \lambda \frac{l}{l_o} + \mu \frac{l^2}{l_o^2} \right\} \quad . \quad . \quad . \quad . \quad (9)$$

For all grains of the first group k becomes unity when $l = l_o$, that is, when the grain is all burned; in this case (9) reduces to

$$1 = \alpha \left(1 + \lambda + \mu\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This relation always subsists between these numerical constants and serves to test the correctness of their derivation in any case.

The following relations which are easily established will be useful:

$$S' = (1 + 2 \lambda + 3 \mu) S_o = S_o + \frac{\alpha V_o}{l_o} (2 \lambda + 3 \mu); \quad (11)$$

or, more generally,

$$S = (1 + 2 \lambda \frac{l}{l_o} + 3 \mu \frac{l^2}{l_o^2}) S_o.$$

We also have

$$\alpha (\lambda + 2 \mu) = \frac{S' l_o}{V_o} - 1.$$

Therefore for all grains whose vanishing surface (S') is zero, we have

$$1 + 2\lambda + 3\mu = 0.$$

and

$$\alpha(\lambda + 2\mu) = -1. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Applications.—We will now apply these formulas to a discussion of various forms of grain now in use or which may come into use.

1. *Sphere.*—For a spherical grain l_o is evidently the radius. Then by mensuration

$$S_o = 4\pi l_o^2$$
$$S' = 0$$
$$V_o = \frac{4\pi l_o^3}{3}$$

Substituting these in (6) and (7) we readily find

$$a = -8\pi l_o \text{ and } b = 4\pi.$$

Therefore from (1)

$$S = 4\pi l_o^2 - 8\pi l_o l + 4\pi l^2 = 4\pi(l_o - l)^2;$$

and, therefore, S is a decreasing function of l.

From (8) we find

$$\alpha = 3, \lambda = -1 \text{ and } \mu = \frac{1}{3};$$

and these substituted in (9) give

$$k = 3\frac{l}{l_o}\left\{1 - \frac{l}{l_o} + \frac{1}{3}\frac{l^2}{l_o^2}\right\} = 1 - \left(1 - \frac{l}{l_o}\right)^3$$

which is the fraction of grain burned in terms of the thickness of the layer l.

If we divide the thickness of web (radius of grain) into five equal parts the following table may be computed, which will be useful for comparing this form of grain with others to be given:

$\frac{l}{l_o}$.	k.	First Differences.
0.0	0.000	.....
0.2	0.488	0.488
0.4	0.784	0.296
0.6	0.936	0.152
0.8	0.992	0.056
1.0	1.000	0.008

The second column gives the entire fraction of grain burned and the third column the fraction of grain burned for each layer. It will be observed that nearly one-half the initial volume of the grain is in the first layer.

2. *Parallelopipedon.*—Let $2\, l_o$ be the least dimension of the parallelopipedon and m and n the other two dimensions. Then, by the rules of mensuration,

$$S_o = 4\, l_o\, m + 4\, l_o\, n + 2\, m\, n$$
$$S' = 2\,(m - 2\, l_o)\,(n - 2\, l_o) = 2\, m\, n - 4\, l_o\, m - 4\, l_o\, n + 8\, l_o^2$$
$$V_o = 2\, l_o\, m\, n$$

Substituting these values of S_o, S' and V_o in (6) and (7), gives

$$a = -8\,(2\, l_o + m + n) \text{ and } b = 24.$$

Making the following substitutions, *viz.*:

$$\frac{2\, l_o}{m} = x \text{ and } \frac{2\, l_o}{n} = y$$

in which x and y are generally less than unity, we have, finally,

$$\alpha = 1 + x + y;\ \lambda = -\frac{x + y + x\,y}{\alpha};\ \mu = \frac{x\,y}{\alpha}.$$

It may be noted that these values of α, λ, μ satisfy equation (10).

There are three special parallelopipedons worthy of separate notice:

(*a*) *Cube.*—The cubical form has been used for ballistite and for some other powders. For this form we evidently have

$$x = y = 1.$$

Therefore

$$\alpha = 3; \lambda = -1; \mu = \frac{1}{3}.$$

These are the same as were found for spherical grains, as might have been inferred. They also apply approximately to sphero-hexagonal, mammoth and rifle powders (old style).

(*b*) *Square Flat Grains.*—For these grains (still used with certain rapid-firing guns), m and n are equal and greater than $2\,l_o$. Therefore x and y are equal and less than unity. Therefore,

$$\alpha = 1 + 2\,x; \lambda = -\frac{x\,(2+x)}{\alpha}; \mu = \frac{x^2}{\alpha}.$$

If these grains are very thin, x becomes a very small fraction and may be omitted in comparison with unity. In this case λ and μ are approximately zero and α unity. This gives

$$k = \frac{l}{l_o}$$

or, a constant emission of gas during the burning; but the grain would be consumed in a very short interval of time.

(*c*) *Grains Made into Long Slender Strips* (*or "Ribbons"*), *with Rectangular Cross-Section.*—These grains are approximately those of the new English powder called "axite." Also of the French "B N" powders, and others. If we suppose the width of the strip to be five times, and the length one-hundred and fifty times, its thickness (which corresponds nearly with the "B N" powders), we shall have $x = \frac{1}{5}$ and $y = \frac{1}{150}$. Therefore

$$\alpha = 1.207; \lambda = -0.172; \mu = 0.001;$$

and the expression for k becomes

$$k = 1.207\frac{l}{l_o}\left\{1 - 0.172\frac{l}{l_o} + 0.001\frac{l^2}{l_o^2}\right\}$$

The following table illustrates the progressiveness of this particular grain:

$\frac{l}{l_o}$.	k.	First Differences.
0.0	0.000	.....
0.2	0.233	0.233
0.4	0.450	0.217
0.6	0.650	0.200
0.8	0.833	0.183
...	.....	0.167
1.0	1.000	1.000

These strips, made up into compact bundles or fagots to form the charge, seem well adapted for rapid-firing guns of moderate caliber. In the application of the expression for k for computing velocities and pressures in the gun, μ may be regarded as zero, and thus greatly shorten the calculations without impairing their accuracy.

If the cross-section of the strip is square, we shall have

$$m = 2\,l_o,\ x = 1 \text{ and } y = \frac{2\,l_o}{n},$$

n being the length of the strip. Therefore, in this case,

$$\alpha = 2 + y;\ \lambda = -\frac{1 + 2\,y}{2 + y};\ \mu = \frac{y}{2 + y}.$$

If the strip be very long in comparison with the linear

dimension of cross-section, y may be considered zero, and we have

$$\alpha = 2; \lambda = -\frac{1}{2}; \mu = 0.$$

Therefore

$$k = 2\frac{l}{l_o}\left\{1 - \frac{1}{2}\frac{l}{l_o}\right\} = 1 - \left(1 - \frac{l}{l_o}\right)^2$$

3. *Solid Cylinder.*—For this form of grain there are two cases to be considered: (*a*) When the diameter of cross-section of the cylinder is the least dimension. (*b*) When the length of the cylinder is the least dimension. That is, a cylinder proper and a circular disk.

(*a*) *Cylinder Proper.*—In accordance with the notation adopted, l_o will be the radius and m the length of the cylinder. We have by mensuration,

$$S_o = 2\pi(l_o m + l_o^2); \; S' = 0; \; V_o = \pi l_o^2 m;$$

whence

$$a = -2\pi(4 l_o + m) \text{ and } b = 6\pi.$$

Putting, as before, $\frac{2 l_o}{m} = x$, there results

$$\alpha = 2 + x; \; \lambda = -\frac{1 + 2x}{2 + x}; \mu = \frac{x}{2 + x}.$$

These are the same expressions for α, λ, μ as was found for a strip with square cross-section, as might readily be inferred.

If x be small in comparison with unity, that is, if the grains are long slender cylinders (thread like), like cordite, we have very approximately,

$$\alpha = 2; \; \lambda = -\frac{1}{2}; \mu = 0;$$

and, as before shown,

$$k = 1 - \left(1 - \frac{l}{l_o}\right)^2$$

The following table was computed by this formula:

$\frac{l}{l_o}$.	k.	First Differences.
0.0	0.00	....
0.2	0.36	0.36
0.4	0.64	0.28
0.6	0.84	0.20
0.8	0.96	0.12
...	....	0.04
1.0	1.00	1.00

Comparing this table with that given for "strips," it will be seen that the burning of cordite is not so progressive as that of axite.

If the length of the solid cylindrical grain be the same as its diameter, then $x = 1$; and we have

$$\alpha = 3; \lambda = -1; \mu = \frac{1}{3}$$

as for spherical and cubical grains.

(*b*) *Circular Disk.*—With this form of grain the *thickness* becomes the least dimension instead of the diameter. Let $2\,l_o$ be the thickness of the disk and R its radius.

Then

$$S_o = 2\pi R\,(2\,l_o + R);\ S' = 2\pi\,(R - l_o)^2;\ V_o = 2\pi\,l_o\,R^2.$$

Whence

$$a = -4\pi\,(2\,R + l_o) \text{ and } b = 6\pi.$$

Therefore making $\frac{2\,l_o}{2\,R} = \frac{l_o}{R} = x$, we have, as has already

been found for square flat grains,

$$\alpha = 1 + 2x; \lambda = -\frac{x(2+x)}{1+2x}; \mu = \frac{x^2}{1+2x}$$

4. *Cylinder with Axial Perforation.*—Let R = radius of grain, r = radius of perforation, and m = its length. We then have

$$2l_o = R - r,$$

and

$$R + r = 2(R - l_o). \quad \therefore R^2 - r^2 = 4l_o(R - l_o).$$

By the rules of mensuration, we find, after reduction,

$$S_o = 2\pi m(R + r) + 2\pi(R^2 - r^2) = 4\pi(m + 2l_o)(R - l_o).$$
$$S' = 4\pi(m - 2l_o)(R - l_o)$$
$$V_o = 4\pi l_o m(R - l_o)$$

Therefore

$$a = -16\pi(R - l_o) \text{ and } b = 0.$$

Making, as before, $x = \frac{2l_o}{m}$ we have

$$\alpha = 1 + x; \lambda = -\frac{x}{1+x}; \mu = 0.$$

Therefore

$$k = (1 + x)\frac{l}{l_o}\left\{1 - \frac{x}{1+x}\frac{l}{l_o}\right\}$$

As an example of this form of grain, suppose the length to be three hundred times the thickness of web. Then

$$x = \frac{1}{300}; \alpha = \frac{301}{300}; \lambda = -\frac{1}{301}; \mu = 0.$$

The expression for k is

$$k = \frac{301}{300}\frac{l}{l_o}\left\{1 - \frac{1}{301}\frac{l}{l_o}\right\} = \frac{l}{l_o}\left\{\frac{301}{300} - \frac{1}{300}\frac{l}{l_o}\right\}$$

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The following table was computed by this formula:

$\frac{l}{l_o}$.	k.	First Differences.
0.0	0.0000	......
0.2	0.2005	0.2005
0.4	0.4008	0.2003
0.6	0.6008	0.2000
0.8	0.8005	0.1997
....	......	0.1995
1.0	1.0000	1.0000

This form of grain is very progressive, much more so than any other form that has been proposed, and seems well adapted for guns of all calibers. The first differences show that for all practical purposes the emission of gas may be considered constant during the entire burning of the grain.

From (11) we have, when $\mu = 0$,

$$S' = (1 + 2\lambda) S_o = \frac{1 - x}{1 + x} S_o.$$

Therefore in this example, when $x = \frac{1}{300}$, we have

$$S' = \frac{299}{301} S_o,$$

and the burning surface during the entire combustion lies between its initial value S_o and its final value $\frac{299}{301} S_o$.

5. Multiperforated Grains.—These grains, which are used exclusively with the heavy artillery of the army and navy of the United States, are cylindrical in form and have seven equal longitudinal perforations, one of which coincides with the axis

of the grain, while the others are disposed symmetrically about the axis, their centres joined forming a regular hexagon. The web thickness ($2\,l_o$) is the distance between any two adjacent circumferences; and therefore, if R is the radius of the grain and r the radius of each of the perforations, we have the relation

$$2\,l_o = \frac{R - 3\,r}{2}$$

From the geometry of the grain as defined above we have the following relations:

$$S_o = 2\,\pi\,\{R^2 - 7\,r^2 + m\,(R + 7\,r)\} \quad . \quad . \quad . \quad . \quad (13)$$

$$S' = S_o + 4\,\pi\,l_o\,(3\,m - 2(R + 7\,r) - 9\,l_o) \quad . \quad (14)$$

$$V_o = \pi\,m\,(R^2 - 7\,r^2) = \frac{m}{2}\{S_o - 2\,\pi\,m\,(R + 7\,r)\} \quad (15)$$

$$V'_o = l_o\,S_o + 2\,\pi\,l_o^2\,(3\,m - 2\,(R + 7\,r) - 6\,l_o) \quad . \quad (16)$$

In these expressions S_o and V_o are the initial surface of combustion and volume, respectively, while S' and V'_o are the vanishing surface and volume burned, when l is about to become l_o and the grain to break up into slivers. If we substitute the values of S_o, S' and V'_o from the above equations in (6) and (7), they reduce to

$$a = 4\,\pi\,(3\,m - 2\,(R + 7\,r))$$

and

$$b = -\,36\,\pi$$

These values of a and b, in equations (8), give

$$\alpha = \frac{R^2 - 7\,r^2 + m\,(R + 7\,r)}{R^2 - 7\,r^2 + (m - 2\,l_o)\,(R + 7\,r + 3\,l_o)} \quad . \quad . \quad (17)$$

$$\lambda = \frac{l_o\,\{3\,m - 2\,(R + 7\,r)\}}{R^2 - 7\,r^2 + m\,(R + 7\,r)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$\mu = -\,\frac{6\,l_o^2}{R^2 - 7\,r^2 + m\,(R + 7\,r)} \quad . \quad . \quad . \quad . \quad (19)$$

These values of α, λ, and μ satisfy the equation of condition

$$\alpha\,(1 + \lambda + \mu) = 1,$$

since when $l = l_o$ the volume V'_o has been consumed. When this occurs, the original form of the grain disappears and there remain twelve slender, three-cornered pieces with curved sides technically called "slivers." These of course must be treated differently. In the applications of these formulas given in Chapter V, the form characteristics of the slivers are assumed to be $\alpha = 2$, $\lambda = \frac{1}{2}$ and $\mu = 0$, with good results.

The form characteristics deduced in (17), (18), and (19), if substituted in (9), will give the fraction of volume V'_o burned. But what is required in practice is the fraction of the entire grain (or charge). This is found by employing V_o instead of V'_o. By this means we find

$$\alpha = 2\, l_o \left\{ \frac{1}{m} + \frac{R + 7\, r}{R^2 - 7\, r^2} \right\}; \quad . \quad . \quad . \quad (20)$$

and this value of α will be used in all the applications. The expressions for λ and μ, being independent of the volume (see equations (8)), are those deduced above.

Substituting the form characteristics in (9) and making

$$l = l_o$$

we shall have the fraction of the entire grain burned when the web thickness is burned. Calling this fraction k' it will be found that

$$k' = \frac{2\, l_o}{m} \left\{ 1 + \frac{(m - 2\, l_o)\,(R + 7\, r + 3\, l_o)}{R^2 - 7\, r^2} \right\} \quad . \quad . \quad (21)$$

This expression for k' would also be obtained by dividing (16) by (15).

It will be seen that for the same web thickness α and k' decrease as m increases, but within moderate limits, their limiting values, when m is infinite, being

$$\alpha = \frac{2\, l_o (R + 7\, r)}{R^2 - 7\, r^2}$$

and

$$k' = \frac{2\,l_o\,(R + 7\,r + 3\,l_o)}{R^2 - 7\,r^2}$$

For the grains employed in the United States service, the ratio $\frac{R}{r}$ varies but little from 11. If we adopt this ratio, the expressions for the form characteristics α, λ, μ and k' become

$$\alpha = \frac{12}{19} + \frac{2\,l_o}{m}$$

$$\lambda = \frac{2\,(m - 6\,l_o)}{19\,l_o + 6\,m}$$

$$\mu = -\frac{4\,l_o}{19\,l_o + 6\,m}$$

$$k' = \frac{16}{19} + \frac{6\,l_o}{19\,m}$$

We also have $R = 5.5\,l_o$ and $r = 0.5\,l_o$.

If, in addition, we make $m = n\,l_o$ we have

$$\alpha = \frac{12}{19} + \frac{2}{n}$$

$$\lambda = \frac{2\,(n - 6)}{6\,n + 19}$$

$$\mu = -\frac{4}{6\,n + 19}$$

$$k' = \frac{16}{19} + \frac{6}{19\,n}$$

It may be noted that the limiting values of these form characteristics, as the length of the grain is indefinitely increased, are,

$$\alpha = \frac{12}{19};\ \lambda = \frac{1}{3};\ \mu = 0 \text{ and } k' = \frac{16}{19}.$$

Also that λ is zero when $n = 6$ and becomes negative when the grain is still further shortened.

It will be seen that the percentage of slivers can never be greater than about 16.

For the grains in use n is approximately 26, which gives

$$\alpha = \frac{175}{247} = 0.70850$$

$$\lambda = \frac{8}{35} = 0.22857$$

$$\mu = -\frac{4}{175} = -0.022857$$

$$k' = \frac{211}{247} = 0.85425$$

There seems to be no valid reason why these, or other simple ratios, for R/r and m/l_o should not be adopted by powder manufacturers for all sizes of m.p. grains, making the diameter of the grain and perforations, and also its length, depend upon the web thickness adopted for a particular gun. For example, the web thickness adopted for the 14-inch gun is 0.1454 inch. Therefore the dimensions of the grains would be

Diameter = 5.5 × 0.1454 = 0.7997 in.
Diameter of perforation = 0.1454/2 = 0.0727 in.
Length = 13 × 0.1454 = 1.89 in.

These dimensions are practically the same as those of the actual grains. From equation (26′) of this chapter it will be seen that the initial surface of one pound of these grains would vary inversely as the web thickness.

For these grains, equations (13) to (16) reduce to

$$S_o = 525\,\pi\,l_o^2$$

$$S' = 729\,\pi\,l_o^2$$

$$V_o = 741\,\pi\,l_o^3$$

$$V'_o = 633\,\pi\,l_o^3$$

The vanishing surface is therefore about 39 per cent. greater than the initial surface.

Captain Hamilton has shown conclusively that the m.p. grains now in use are much too short to secure a proper alignment in the powder chamber, and that this lack of alignment conduces to excessive pressure.*

If we make $n = 200$, that is, make the length of the grains 100 times the web thickness, we should have

$$\alpha = 0.64158$$
$$\lambda = 0.31829$$
$$\mu = -0.00328$$
$$k' = 0.84368$$

This value of n would make the length of the grains for the 14-inch gun 14.52 inches; which would not only secure a good alignment of the grains in the containing bag, but would also give a much less initial surface of combustion to the charge and would thus reduce the maximum pressure.

The general expression for the surface of combustion of an m.p. grain with 7 perforations, in terms of the thickness of web burned, is by (1),

$$S = S_o + 4\pi(3m - 2(R + 7r))l - 36\pi l^2$$

Differentiating twice, we have

$$\frac{ds}{dl} = 4\pi(3m - 2(R + 7r)) - 72\pi l$$

$$\frac{d^2 s}{dl^2} = -72\pi$$

There is, therefore, a maximum value of S which occurs when

$$l = \frac{3m - 2(R + 7r)}{18}$$

* Journal U. S. Artillery, July–August, 1908, page 9.

and the maximum surface of combustion is

$$S_{max.} = S_o + \frac{\pi\,(3\,m - 2\,(R + 7\,r))^2}{9}$$

From these formulas are easily deduced the following:

1. When $3\,m - 2\,(R + 7\,r) = 0$, S is a decreasing function of l during the entire burning of the web thickness.

2. When $3\,m - 2\,(R + 7\,r)$ is equal to, or greater than, $18\,l_o$ the grain burns with an increasing surface.

3. When $3\,m - 2\,(R + 7\,r)$ lies between 0 and $18\,l_o$ the surface of combustion is at first increasing and then decreasing.

Expression for Weight of Charge Burned.—If we assume that the entire charge is ignited at the same instant, which is practically the case with an igniter at both ends of the cartridge, the combustion of the charge will be expressed by the same function that applies to a single grain. Therefore if y is the weight of the charge burned at any period of the combustion and $\tilde{\omega}$ the weight of the entire charge, we may assume the equality (since the weights are proportional to the volumes)

$$k = \frac{y}{\tilde{\omega}} = \alpha\frac{l}{l_o}\left\{1 + \lambda\frac{l}{l_o} + \mu\frac{l^2}{l_o^2}\right\} \quad . \quad . \quad . \quad (22)$$

In this equation α is always positive from its definition, *viz.*: $\alpha = \frac{S_o\,l_o}{V_o}$. It varies in value from 3 (spheres and cubes) to less than unity (service multiperforated grains). The smaller α is, *cæteris paribus*, the less will be the maximum pressure for a given charge. Of the other characteristics, λ and μ, either may be positive, negative, or zero, but not both at the same time.

Expressions for Initial Volume and Surface of Combustion of a Charge of Powder.—Let N be the number of grains in unit weight of powder, V' the volume of unit weight of water, and δ the specific gravity of the powder. Then, from the definition of specific gravity,

$$\delta = \frac{V'}{N V_o}, \quad \ldots \ldots \quad (23)$$

since we may assume that the weights are proportional to the volumes. The number of grains in unit weight of powder can be counted, and, with the carefully moulded grains now in use, V_o can be calculated with great accuracy. Thus (23) can be employed to determine the specific gravity of a powder when it is not given by the manufacturer, as is usually the case. For the large grains designed for seacoast guns the number of grains in 100 units should be counted, estimating the fraction of a grain in excess. For small-arms powder, if the specific gravity of the mass of which the grains are made is known, the number of grains in unit weight may be computed by the formula

$$N = \frac{V'}{\delta V_o}. \quad \ldots \ldots \quad (24)$$

The units to be used in these and other formulas that will be deduced will be considered later.

Initial Surface of Unit Weight of Powder and of the Entire Charge.—Let S_1 be the initial surface of the grains of unit weight of powder. Then if S_o is the surface of one grain, we have, by (24)

$$S_1 = N S_o = \frac{V' S_o}{\delta V_o}. \quad \ldots \ldots \quad (25)$$

But by (8)

$$S_o = \frac{\alpha V_o}{l_o}$$

Therefore

$$S_1 = \frac{\alpha V'}{\delta l_o} = \frac{\alpha N V_o}{l_o}, \quad \ldots \ldots \quad (26)$$

for one unit weight of powder; and for $\tilde{\omega}$ units weight,

$$S_{\tilde{\omega}} = \frac{\alpha \tilde{\omega} V'}{\delta l_o} = \frac{\alpha \tilde{\omega} N V_o}{l_o} \quad \ldots \ldots \quad (26')$$

This simple formula was first published in the *Journal U. S. Artillery* for November-December, 1905. It shows that for two charges of equal weight and made up of grains of the same density and thickness of web, but of dissimilar forms, the entire surfaces of all the grains in the two charges are proportional to the corresponding values of α. It also shows that if the initial surfaces of two charges of equal weight but made up of grains of dissimilar forms, are to be the same, the web thicknesses must be inversely as the values of α. For example, if the two charges are made up, the one of cubes and the other of long slender cylinders (axite and cordite), the web thickness of the former must be one-half greater than the latter to obtain the same initial surface for each charge. These principles are important since the maximum pressure in a gun varies very nearly with the initial surface of the charge.

Volume of Entire Charge.—Let $V_{\bar{\omega}}$ be the volume of a charge of $\bar{\omega}$ units weight supposed to be reduced to a single homogeneous grain. For a single grain of unit weight (23) gives

$$V_o = \frac{V'}{\delta}$$

and for $\bar{\omega}$ units

$$V_{\bar{\omega}} = \frac{\bar{\omega}\, V'}{\delta} \qquad (27)$$

Gravimetric Density.—Gravimetric density is the density of a charge of powder when the spaces between the grains are considered. It is measured by the ratio of the weight of any given volume of the powder grains to the weight of the same volume of water. Since one pound of water fills 27.68 cubic inches we may say that the gravimetric density of a powder is the weight in pounds of 27.68 cubic inches of the powder not pressed together except by its own weight. Or, if we take a cubic foot as the unit and designate the gravimetric density by γ, the weight of a cubic foot of the powder grains by $\bar{\omega}'$, and by w the weight of

a cubic foot of water, we shall have by definition,

$$\gamma = \frac{\bar{\omega}'}{w} = \frac{\bar{\omega}'}{1728/27.68} = \frac{\bar{\omega}'}{62.427}.$$

It is evident that γ will vary not only with the density of the individual grains but also with the volume of the interstices between them; and this latter varies with the general form of the grains, or, in other words, with their ability to pack closely or the reverse. It is evident that the maximum value of γ is the weight of a cubic foot of solid powder, in which case the above ratio would be the specific gravity of the powder, designated by δ. The gravimetric density is therefore always less than the specific gravity. For modern powders gravimetric density is of very little importance.

Density of Loading.—Density of loading is defined to be the "ratio of the weight of charge to the weight of a volume of water just sufficient to fill the powder chamber." Let Δ be the density of loading and V_c the volume of the powder chamber. Since V' is the volume of unit weight of water it is evident that V_c/V' is the weight of a volume of water equal to the volume of the chamber. Hence by definition,

$$\Delta = \frac{\bar{\omega} V'}{V_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

From (27) we have,

$$\bar{\omega} = \frac{\delta V_{\bar{\omega}}}{V'}$$

and this substituted in (28), gives

$$\Delta = \frac{\delta V_{\bar{\omega}}}{V_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

From this last equation the density of loading may be defined as the ratio of the volume of the powder grains supposed to be reduced to a single grain, to the volume of the chamber, multiplied by the density of the powder. If $V_{\bar{\omega}} = V_c$, that is, if the

chamber is filled by a single grain, then $\Delta = \delta$; and this is the superior limit of density of loading. The inferior limit is, of course, zero, namely, when $V_{\bar{\omega}} = 0$. If the density of loading is unity it follows from (28), that

$$\bar{\omega} = \frac{V_c}{V'};$$

that is, the weight of charge equals the weight of water that would fill the chamber.

Reduced Length of Initial Air Space.—By initial air space is meant that portion of the volume of the chamber not occupied by the powder grains constituting the charge. The reduced length of the initial air space is the length of a cylinder whose cross-section is the same as that of the bore, and whose volume is equal to the initial air space. Denote this length by z_o and the area of cross-section of the bore by ω. Then as $V_c - V_{\bar{\omega}}$ is the volume of the air space we have

$$z_o = \frac{V_c - V_{\bar{\omega}}}{\omega}$$

Substituting for V_c and $V_{\bar{\omega}}$ their values from (28) and (27), we have

$$z_o = \frac{\bar{\omega} V'}{\omega}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right).$$

Put

$$a = \frac{1}{\Delta} - \frac{1}{\delta} = \frac{\delta - \Delta}{\Delta \delta}.$$

Then

$$z_o = \frac{a \bar{\omega} V'}{\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Working Formulas for English and French Units.—The English units used with formulas (23) to (30), inclusive, are the pound and inch. Therefore

$$V' = 27.68 \text{ cubic inches, nearly.}$$

The French units employed with the same formulas are the kilogramme and decimetre. For these units we have

$$V' = 1 \text{ cubic decimetre.}$$

The two sets of formulas in working form are therefore:—

English Units		French Units	
$\delta = \dfrac{27.68}{N V_o}$	(23′)	$\delta = \dfrac{1}{N V_o}$	(23″)
$N = \dfrac{27.68}{\delta V_o}$	(24′)	$N = \dfrac{1}{\delta V_o}$	(24″)
$S_{\tilde{\omega}} = \dfrac{27.68\, \alpha\, \tilde{\omega}}{\delta\, l_o}$	(26′)	$S_{\tilde{\omega}} = \dfrac{\alpha\, \tilde{\omega}}{\delta\, l_o}$	(26″)
$V_{\tilde{\omega}} = \dfrac{27.68\, \tilde{\omega}}{\delta}$	(27′)	$V_{\tilde{\omega}} = \dfrac{\tilde{\omega}}{\delta}$	(27″)
$\Delta = \dfrac{27.68\, \tilde{\omega}}{V_c}$	(28′)	$\Delta = \dfrac{\tilde{\omega}}{V_c}$	(28″)
$z_o = \dfrac{27.68\, a\, \tilde{\omega}}{\omega}$	(30′)	$z_o = \dfrac{a\, \tilde{\omega}}{\omega}$	(30″)

EXAMPLES

1. Compute the number of grains in a pound of the powder used with the service magazine rifle. Also the initial surface.

The grains of this powder are pierced cylinders of the following dimensions:

$$R = 0.''045$$
$$r = 0.015 \qquad \therefore \quad 2\, l_o = 0.''03$$
$$m = \frac{1}{21} \text{ in.}$$
$$\delta = 1.65$$
$$\alpha = 1.63$$
$$\tilde{\omega} = 1$$

From (24′) and (26′), we have

$$N = \frac{27.68}{4\pi l_o m (R - l_o)\delta} = 62300$$

$$S_1 = \frac{27.68 \times 1.63}{1.65 \times 0.015} = 1823 \text{ in.}^2$$

It is officially stated that the number of grains per pound varies from 83,000 to 93,000. This discrepancy is partly due to shrinkage and partly to the breaking and chipping of the grains. Possibly also to the method of counting.

2. What is the entire initial surface of a charge of 70 lbs. of the m.p. powder designed for the 8-inch rifle? For this powder we have

$$R = 0''.256; \; r = 0.''.0255; \; m = 1''.029; \; \delta = 1.58$$

$$l_o = \frac{R - 3r}{4} = 0''.044875; \; \alpha = 0.72667$$

$$\therefore S_{\hat{\omega}} = \frac{27.68 \times 0.72667 \times 70}{1.58 \times 0.044875} = 19813 \text{ in.}^2$$

3. Suppose the powder of example 2 to be made into cubes having the same thickness of web. What would be the initial surface of the charge?

For a cube $\alpha = 3$. Therefore

$$S_{\hat{\omega}} = \frac{3 \times 19813}{0.72667} = 81795 \text{ in.}^2$$

To make the initial surface of the latter charge the same as the former the web thickness would have to be

$$2\,l_o = \frac{3 \times 0.08975}{0.72667} = 0''.37$$

4. The volume of the chamber of the 12-inch rifle is 17487 cubic inches. If the charge is 400 lbs. what is the density of loading? Ans.: $\Delta = 0.633$.

CHAPTER IV

COMBUSTION AND WORK OF A CHARGE OF POWDER IN A GUN

It has been established by experiment that a grain of modern powder burns in concentric, parallel layers, and that the velocity of combustion under constant pressure is uniform. Let l_o be one-half the web thickness of a grain and τ the time of burning this thickness under the constant pressure of the atmosphere. We then have, since the web burns on both sides,

$$\frac{l_o}{\tau} = \text{velocity of combustion} = \text{constant} = v_c \text{ (say)}. \qquad (1)$$

In the bore of a gun, however, the pressure surrounding the grain is very far from being constant and greatly exceeds the atmospheric pressure. All writers on interior ballistics agree that the velocity of combustion may be regarded at each instant as proportional to some power of the pressure; but they differ widely among themselves as to what this power is. Sainte-Robert, Vieille, Gossot, and Liouville give reasons (based, however, upon experiments made with a small quantity of powder exploded in an eprouvette of a few cubic inches capacity) for adopting the exponent $\frac{2}{3}$. Centervall makes the exponent $\frac{9}{10}$ for "Nobel N K" powder. Sebert and Hugoniot, from observations of the recoil of a 10-cm. gun mounted on a free-recoil carriage, deduced a law of burning directly proportional to the pressure. This law is the most simple of all and allows an easy and complete integration of the equations entering into the problem.* But simple as is this law of Sebert and Hugoniot,

* See Journal U. S. Artillery, vol. 7, pp. 62–82.

we prefer to make use of Sarrau's law of the square root of the pressure, because the resulting formulas are easily worked and give results which "agree very well with facts" as stated by Sarrau, and as has been repeatedly shown by the writer and others.

Sarrau's law of burning under a variable pressure p leads directly to the equation,

$$\frac{d\,l}{d\,t} = \frac{l_o}{\tau}\left(\frac{p}{p_o}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which p_o is the atmospheric pressure and l the thickness of layer burned in time t.

It will be assumed that the variable pressure p in the bore is measured by the energy of translation imparted to the projectile (which is many times the sum of all the other energies entering into the problem); and it will be taken for granted that all the other work done by the expansion of the powder gas may be accounted for by giving suitable values to the constants so as to satisfy the firing data by means of which they are determined. This procedure will be fully illustrated further on.

If p is the variable pressure per unit of surface upon the base of the projectile at any instant, ω the area of the base, and u the corresponding distance travelled by the projectile from its firing seat, we have from the principle of energy and work,

$$\omega\,p = \frac{w}{g}\,\frac{d^2\,u}{d\,t^2}$$

in which w is the weight of the projectile.

But from mechanics and calculus,

$$\frac{d^2 u}{d\,t^2} = \frac{d\,v}{d\,t} = \frac{d\,v}{d\,u}\cdot\frac{d\,u}{d\,t} = v\frac{d\,v}{d\,u} = \frac{1}{2}\,\frac{d\,(v^2)}{du},$$

in which v from now on represents velocity.

Therefore

$$\omega\,p = \frac{w}{2\,g}\;\frac{d\,(v^2)}{d\,u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Combining (2) and (3), we have

$$\frac{d\,l}{d\,t} = \frac{l_o}{\tau}\left(\frac{w}{2\,g\,\omega\,p_o}\right)^{\frac{1}{2}} \left(\frac{d\,(v^2)}{d\,u}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (4)$$

Since z_o is the reduced length of the initial air-space and u the distance travelled by the projectile from its firing seat, we may say very approximately, by the principle of the covolume, that u/z_o is the number of volumes of expansion of the gas during the travel u, and this whether the charge is all converted into gas or not. If we make $u/z_o = x$, and therefore $d\,u/d\,x = z_o$, (3) and (4) become

$$\omega\,p = \frac{w}{2\,g\,z_o} \cdot \frac{d\,(v^2)}{d\,x} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\frac{d\,l}{d\,t} = \frac{l_o}{\tau}\left(\frac{w}{2\,g\,\omega\,p_o\,z_o}\right)^{\frac{1}{2}} \left(\frac{d\,(v^2)}{d\,x}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (6)$$

It will be seen that (6) connects the velocity of burning of the grain with the velocity of travel of the projectile in the bore. It will be better to make x the independent variable in the first member as well as the second.

We have from calculus,

$$\frac{d\,l}{d\,t} = \frac{d\,l}{d\,x}\;\frac{d\,x}{d\,u}\;\frac{d\,u}{dt} = \frac{v}{z_o} \cdot \frac{d\,l}{d\,x}$$

Therefore, substituting in (6),

$$\frac{d\,l}{d\,x} = \frac{l_o}{\tau}\left(\frac{w\,z_o}{2\,g\,\omega\,p_o}\right)^{\frac{1}{2}} \left(\frac{d\,(v^2)}{d\,x}\right)^{\frac{1}{2}} \frac{1}{v}$$

Integrating between the limits o and x, we have

$$\frac{l}{l_o} = \frac{1}{\tau}\left(\frac{w\,z_o}{2\,g\,\omega\,p_o}\right)^{\frac{1}{2}} \int_o^x \left(\frac{d\,(v^2)}{d\,x}\right)^{\frac{1}{2}} \frac{1}{v}\,d\,x \quad . \quad . \quad (7)$$

In order to perform the integration indicated, we must know the relation existing between v and x, that is between the velocity of the projectile in the bore at any instant and the corresponding

number of volumes of expansion of the gas. We get this relation from (19) Chapter II, which is

$$v^2 = \frac{6\,g\,f\,y}{w}\left\{ 1 - \frac{1}{(1+x)^{\frac{1}{3}}} \right\} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From this equation we deduce by simple differentiation

$$\frac{d\,(v^2)}{d\,x} \cdot \frac{1}{v} = \frac{1}{\sqrt{3}\,(1+x)^{\frac{1}{2}}\sqrt{(1+x)^{\frac{1}{3}} - 1}}$$

Substituting this in (7) and making

$$X_o = \int^x \frac{d\,x}{\sqrt{(1+x)^{\frac{1}{2}}}\sqrt{(1+x)^{\frac{1}{3}} - 1}}, \quad . \quad . \quad . \quad (9)$$

we have

$$\frac{l}{l_o} = \frac{1}{\tau}\left(\frac{w\,z_o}{6\,g\,\omega\,p_o}\right)^{\frac{1}{2}} X_o \,. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It will be observed that X_o is a function of a ratio and is independent of any unit, and may therefore be tabulated with x as the argument.

If we put

$$K = \frac{1}{\tau}\left(\frac{w\,z_o}{6\,g\,\omega\,p_o}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

we have

$$\frac{l}{l_o} = K\,X_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Substituting the value of l/l_o from (12) in (22), Chapter III, we have

$$k = \frac{y}{\tilde{\omega}} = \alpha\,K\,X_o\,(1 + \lambda\,K\,X_o + \mu\,(K\,X_o)^2) \quad . \quad (13)$$

an equation which gives the fraction of the charge burned at any instant in terms of the volumes of expansion of the gases generated. When the powder is all burned in the gun (if it be all burned before the projectile leaves the bore), we have $y = \tilde{\omega}$ and $l = l_o$.

If, therefore, we distinguish X_o by a dash when $l = l_o$, (12) becomes

$$K \overline{X}_o = 1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and (13) reduces to

$$1 = \alpha (1 + \lambda + \mu)$$

a fundamental relation established in Chapter III.

Substituting the value of K from (14) in (13), we have, while the powder is burning, the relation

$$\frac{y}{\tilde{\omega}} = \alpha \frac{X_o}{\overline{X}_o} \left\{ 1 + \lambda \frac{X_o}{\overline{X}_o} + \mu \left(\frac{X_o}{\overline{X}_o} \right)^2 \right\} \quad . \quad . \quad . \quad (15)$$

Expression for Velocity of Projectile while the Powder is Burning.—Substituting the value of y from (15) in (8) and making

$$X_1 = X_o \left(1 - \frac{1}{(1 + x)^{\frac{1}{3}}} \right) \quad . \quad . \quad . \quad . \quad (16)$$

we have

$$v^2 = 6 \, g \, \alpha \, f \frac{\tilde{\omega}}{w} \cdot \frac{X_1}{\overline{X}_o} \left\{ 1 + \lambda \frac{X_o}{\overline{X}_o} + \mu \left(\frac{X_o}{\overline{X}_o} \right)^2 \right\} \quad . \quad . \quad (17)$$

This equation holds only while the powder is burning and ceases to be true when $X_o > \overline{X}_o$.

Velocity of Projectile when y $= \tilde{\omega}$. When $X_o = \overline{X}_o$ and, therefore, $X_1 = \overline{X}_1$, equation (17) reduces to

$$\overline{v}^2 = 6 \, g \, f \frac{\tilde{\omega} \, \overline{X}_1}{w \, \overline{X}_2} \, \alpha \, (1 + \lambda + \mu);$$

or, since

$$\alpha \, (1 + \lambda + \mu) = 1,$$

it becomes

$$\overline{v}^2 = 6 \, g \, f \frac{\tilde{\omega} \, \overline{X}_1}{w \, \overline{X}_o}$$

This equation is, of course, the same as (8) from which it is

derived as is evident from (16). Putting

$$\frac{\overline{X}_1}{\overline{\overline{X}}_o} = \overline{X}_2,$$

or, generally,

$$\frac{X_1}{X_o} = X_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

the expression for $\overline{v}^2$ becomes

$$\overline{v}^2 = 6\,g\,f\,\frac{\tilde{\omega}}{w}\,\overline{X}_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

It should be remembered that all symbols employed in this work affected with a dash refer to the position of the projectile, either in the bore on in the bore prolonged, when the powder has all been burned, and therefore where $y = \tilde{\omega}$.

From (19), we have

$$6\,g\,f = \frac{\overline{v}^2}{\overline{\overline{X}}_2}\frac{w}{\tilde{\omega}}\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and this substituted in (17), gives

$$v^2 = \alpha\,\overline{v}^2\,\frac{X_1}{\overline{\overline{X}}_1}\left\{ 1 + \lambda\,\frac{X_o}{\overline{\overline{X}}_o} + \mu\left(\frac{X_o}{\overline{\overline{X}}_o}\right)^2 \right\} \quad . \quad . \quad (21)$$

For convenience, put

$$M = \frac{\alpha\,\overline{v}^2}{\overline{\overline{X}}_1};\; N = \frac{\lambda}{\overline{\overline{X}}_o};\; N' = \frac{\mu}{\overline{\overline{X}}^2_o} = \frac{\mu}{\lambda^2}N^2.$$

Then, finally, while the powder is burning,

$$v^2 = M\,X_1\left\{ 1 + N\,X_o + N'\,X_o^2 \right\} \quad . \quad . \quad . \quad . \quad (22)$$

Velocity of Projectile after Powder is all Burned.—The velocity of the projectile after the powder is all burned is given by (8), substituting $\tilde{\omega}$ for y. Reducing by means of (16), (18), and (20), and denoting velocity after the powder is all burned by capital V, equation (8) becomes

$$V^2 = \overline{v}^2\,\frac{X_2}{\overline{\overline{X}}_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The velocity then after the powder is all burned varies directly as the square root of X_2. From (16) and (18), we have

$$X_2 = 1 - \frac{1}{(1 + x)^{\frac{1}{3}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and therefore the superior limit as x (or u) increases indefinitely is unity. On this supposition (23) becomes

$$V^2 = \frac{\bar{v}^2}{\bar{X}_2} = V_1^2 \text{ (say)} \quad . \quad . \quad . \quad . \quad (25)$$

We may regard V_1 then as the theoretical limiting velocity after an infinite travel. In terms of V_1 (23) becomes

$$V^2 = V_1^2 X_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Since from (25) $\bar{v}^2 = \bar{X}_2 V_1^2$, therefore

$$\frac{\bar{v}^2}{\bar{X}_1} = \frac{\bar{X}_2 V_1^2}{\bar{X}_1} = \frac{V_1^2}{\bar{X}_o} \text{ (by (18))}$$

and, therefore,

$$M = \frac{\alpha V_1^2}{\bar{X}_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Pressure on Base of Projectile while Powder is Burning.— Differentiating (17) with respect to the independent variable x and putting for simplicity

$$X_3 = \frac{d X_1}{d x}; \ X_4 = X_o + \frac{X_1}{X_3} \frac{d X_o}{d x}; \ X_5 = X_o^2 + \frac{2 X_o X_1}{X_3} \frac{d X_o}{d x},$$

we have

$$\frac{d (v^2)}{d x} = M X_3 \{1 + N X_4 + N' X_5\}.$$

Therefore, from (5)

$$p = \frac{w}{2 g \omega z_o} M X_3 \{1 + N X_4 + N' X_5\}.$$

Combining the constants outside the brackets into one multiplier by making

$$\frac{w M}{2 g \omega z_o} = M', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

we have the following expression for the pressure per unit of surface, on the base of the projectile:—

$$p = M' X_3 \{ 1 + N X_4 + N' X_5 \} \quad . \quad . \quad . \quad (29)$$

Pressure after the Powder is all Burned.—Differentiating (26) with reference to x and substituting the differential coefficient in (5) we have, employing capital P to express pressure in this case,

$$P = \frac{w V_1^2}{2 g \omega z_0} \frac{d X_2}{d x}.$$

But from (24)

$$\frac{d X_2}{d x} = \frac{1}{3 (1 + x)^{\frac{4}{3}}};$$

whence, putting

$$\frac{w V_1^2}{6 g \omega z_0} = P' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

we have finally

$$P = \frac{P'}{(1 + x)^{\frac{4}{3}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

If we make $x = 0$ in (31), we have

$$P = P'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Therefore P' is the pressure per unit of surface at the origin supposing the powder to be all burned before the projectile moves from its seat.

Relation Between f and P′.—From (19) and (25) we get

$$f = \frac{w V_1^2}{6 g \bar{\omega}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Combining this with (30) there results

$$P' = \frac{\bar{\omega} f}{z_0 \omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Since f is (at least theoretically) the pressure per unit of surface of the gases of one pound of powder at temperature of combustion, occupying unit volume, it follows from (34) that

P' is the pressure per unit of surface of the gases of $\bar{\omega}$ pounds of powder (the entire charge), occupying a volume equal to the initial air space $z_o \omega$, as has already been shown by equation (32). Equation (31) is, therefore, the equation of the pressure curve upon the supposition that the charge is all converted into gas before the projectile has moved from its seat. From equation (30′), Chapter III, we have,

$$z_o \omega = \frac{27.68}{1728} a \bar{\omega} \text{ cubic feet.}$$

Therefore, from (34)

$$P' = \frac{1728}{27.68} \frac{f}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

Values of the X Functions.—These values may be most easily and simply expressed by means of auxiliary circular functions. Thus let

$$(1 + x)^{\frac{1}{6}} = \sec \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Then, by trigonometry,

$$\sin^2 \phi = 1 - \frac{1}{(1 + x)^{\frac{1}{3}}} = X_2. \quad \text{(from (22))}$$

and

$$\tan \phi = \sqrt{(1 + x)^{\frac{1}{3}} - 1}$$

Also

$$d x = 6 \sec^6 \phi \tan \phi \, d \phi$$

Substituting these values in the expression for X_o we have

$$X_o = 6 \int_o^\phi \sec^3 \phi \, d \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Integrating, we have

$$X_o = 3 \sec \phi \tan \phi + 3 \log_e (\sec \phi + \tan \phi) \quad . \quad (38)$$

By substituting the values of $\sec \phi$ and $\tan \phi$ given above in (38), we get an expression for X_o in terms of x. But it is of no practical interest. The definite integral in (37) is a well-known function of ϕ and has been extensively used in exterior ballistics.

A table of this function computed for every minute of arc up to 87° was first published by Euler, and has recently been reprinted (1904) at the Government Printing Office and issued as "Supplement No. 2, to Artillery Circular M." By means of this table it is easy to compute X_o for any value of x. First compute ϕ by (36), and then take the definite integral corresponding to ϕ, which has been symbolized by (ϕ), from the table just mentioned. We then have

$$X_o = 6\,(\phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

Since

$$X_2 = \sin^2\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

we have from (18)

$$X_1 = X_o \sin^2\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

By definition

$$X_3 = \frac{d\,X_1}{d\,x}$$

But from (41)

$$\frac{d\,X_1}{d\,x} = \frac{d\,X_o}{d\,x}\sin^2\phi + 2\,X_o\sin\phi\cos\phi\,\frac{d\phi}{d\,x}$$

From (9) and (36) we deduce

$$\frac{d\,X_o}{d\,x}\sin^2\phi = \sin\phi\cos^4\phi.$$

Also, from what precedes,

$$2\sin\phi\cos\phi\,\frac{d\phi}{d\,x} = \frac{\sin\phi\cos\phi}{3\sec^6\phi\tan\phi} = \frac{\cos^8\phi}{3}.$$

Therefore

$$X_3 = \sin\phi\cos^4\phi + \frac{1}{3}X_o\cos^8\phi.$$

Let

$$X = \frac{1}{1 + \frac{1}{3}X_o\cos^4\phi\operatorname{cosec}\phi}$$

Then we have

$$X_3 = \frac{\sin\phi\cos^4\phi}{X} \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

From the foregoing equations we find

$$\frac{X_1}{X_3}\frac{d\,X_o}{d\,x} = X\,X_o$$

by means of which are easily deduced from the definitions of X_4 and X_5 the following simple equations:—

$$X_4 = X_o\,(1 + X) \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

and

$$X_5 = X_o^2\,(1 + 2\,X) \quad . \quad . \quad . \quad . \quad . \quad (44)$$

By means of equations (39), (40), (41), (42), (43) and (44), the table of the logarithms of the X-functions given at the end of the volume was computed.

Some Special Formulas. Dividing (21) by (15) and reducing by (25), we have, since $y/\bar{\omega} = k$,

$$v^2 = k\,V^2 = k\,V_1^2 X_2 \quad . \quad . \quad . \quad . \quad . \quad (45)$$

That is, the velocity of the projectile at any travel before the charge is all burned is equal to what the velocity would have been at the same travel had all the charge been converted into gas before the projectile moved, multiplied into the square root of the fraction of charge burned.

For spherical, cubical, and certain other forms of grain, we have $\alpha = 3$, $\lambda = -1$ and $\mu = \frac{1}{3}$. Substituting these in (15), we have by obvious reductions,

$$k = 1 - \left(1 - \frac{X_o}{\overline{X}_o}\right)^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

and therefore

$$X_o = \overline{X}_o\left\{1 - (1 - k)^{\frac{1}{3}}\right\} \quad . \quad . \quad . \quad . \quad . \quad (47)$$

For cordite and similar grains we have $\alpha = 2$, $\lambda = -\frac{1}{2}$ and $\mu = 0$. Substituting these in (15), gives

$$k = 1 - \left(1 - \frac{X_o}{\overline{X}_o}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad (48)$$

and

$$X_o = \overline{X}_o\left\{1 - (1 - k)^{\frac{1}{2}}\right\} \quad . \quad . \quad . \quad . \quad . \quad (49)$$

Equations (46) and (48) give the fraction of the charge consumed for any given travel of the projectile, and, conversely, (47) and (49) enable us to determine the travel of projectile for any given fraction of charge burned. For any other forms of grain the solution of a complete cubic equation is necessary to determine X_o when k is given. See equation (15).

Expressions for Maximum Pressure.—It is well known that the maximum pressure in a gun occurs when the projectile has moved but a comparatively short distance from its seat, or when u and x are relatively small. The position of maximum pressure is not fixed but varies with the resistance encountered. As a rule it will be found that the less the resistance to be overcome by the expanding gases the sooner will they exert their maximum pressure, and the less will the maximum pressure be. The differentiation of (29) gives an analytical expression for the maximum value of p; but it is too complicated to be of any practical use. A reference to the table of the X functions shows that X_3 is approximately a maximum when $x = 0.64$, while X_4 and X_5 increase indefinitely. When λ is negative it is evident that p is a maximum when x is less than 0.64; and when λ is positive, when x is greater than 0.64. Therefore there will be two cases depending upon whether the grains burn with an increasing or a decreasing surface. These will be considered separately.

(a) *When the grains burn with a decreasing surface; or what is the same thing, when λ is negative.* A function at, or near, its maximum changes its value slowly. Therefore a moderate

variation of the position of maximum pressure will have no practical effect upon its computed value. It has been found by trial in numerous cases that $x = 0.45$ gives the position of maximum pressure when λ is negative with great precision. For this value of x the table gives,

$$\log X_3 = 9.85640 - 10$$
$$\log X_4 = 0.48444.$$
$$\log X_5 = 0.93587.$$

Substituting these in (29) and designating the maximum pressure by p_m, we have approximately, when λ is negative,

$$p_m = [9.85640 - 10]\, M' \{1 - [0.48444]\, N + [0.93587]\, N'\} \quad (50)$$

or,

$$p_m = 0.71846\, M' \{1 - 3.0510\, N + 8.6273\, N'\} \quad . \quad (50')$$

(b) *When the grains burn with an increasing surface.* When the grains burn with an increasing surface λ is generally positive, and it will not be far wrong to assume that the maximum pressure occurs when $x = 0.8$. For this value of x the table gives:

$$\log X_3 = 9.86027 - 10.$$
$$\log X_4 = 0.60479$$
$$\log X_5 = 1.17352$$

Substituting these in (29), we have,

$$p_m = [9.86027 - 10]\, M' \{1 + [0.60479]\, N - [1.17352]\, N'\} \quad (51)$$

or,

$$p_m = 0.72489\, M' \{1 + 4.0252\, N - 14.911\, N'\} \quad . \quad (51')$$

Expressions for Computing τ and the Velocity of Combustion. From (11) and (14) we have

$$\tau = \left(\frac{w\ z_0}{6\, g\, \omega\, p_o}\right)^{\frac{1}{2}} \overline{X}_o \quad . \quad . \quad . \quad . \quad (52)$$

If v_c is the velocity of combustion under atmospheric pressure we shall have

$$v_c = \frac{l_o}{\tau};$$

and therefore

$$v_c = \left(\frac{6\,g\,\omega\,p_o}{w\,z_o}\right)^{\frac{1}{2}} \frac{l_o}{\overline{\overline{X}}_o} \quad . \quad . \quad . \quad . \quad (53)$$

Let v'_c be the velocity of combustion at any instant under the varying pressure p. Then from (2) we have

$$v'_c = v_c \left(\frac{p}{p_o}\right)^{\frac{1}{2}} = \left(\frac{6\,g\,\omega\,p}{w\,z_o}\right)^{\frac{1}{2}} \frac{l_o}{\overline{\overline{X}}_o} \quad . \quad . \quad . \quad (53')$$

Working Formulas. English Units.—It is customary in our service, following the English practice, to express the volumes of the powder chamber and bore in cubic inches; the various pressures in pounds per square inch; the caliber, reduced length of initial air space, and travel of the projectile in the bore, in inches; while the velocity of the projectile is expressed in foot-seconds and its weight in pounds and ounces. These units are apt to cause confusion and error in the applications of ballistic formulas; and to avoid this as much as possible it will be well to reproduce the most important of the formulas deduced in the preceding pages with all the reductions made and the mathematical and physical constants introduced and combined into one numerical coefficient. The physical constants adopted for English units (foot-pound), are the following:

$g = 32.16$ f.s. (mean for the United States)
$p_o = 14.6967$ lbs. per in.^{2}
$V' = 27.68$ cubic inches.

f is taken in pounds per square inch. The formulas are renumbered for convenience.

$$\Delta = 27.68 \frac{\tilde{\omega}}{V_c} = [1.44217] \frac{\tilde{\omega}}{V_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

$$a = \frac{1}{\Delta} - \frac{1}{\delta} = \frac{\delta - \Delta}{\Delta\,\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

$$z_o = \frac{4 \times 27.68}{\pi} \cdot \frac{a\,\tilde{\omega}}{d^2} = [1.54708] \frac{a\,\tilde{\omega}}{d^2} \text{ (inches)} \quad . \quad (56)$$

$$x = \frac{u}{z_o} \qquad (57)$$

$$V_1^2 = 144 \times 6\, g\, \frac{f\,\bar{\omega}}{w} = [4.44383]\frac{f\,\bar{\omega}}{w} \text{ (foot-seconds)} \qquad (58)$$

$$V_1^2 = \frac{v^2}{k\,X_2} = \frac{V^2}{X_2} = \frac{M\,\overline{X}_o}{\alpha} = \frac{\lambda\,M}{\alpha\,N} = \frac{3\,M\,P'}{M'} = \frac{\overline{v}^2}{\overline{X}_2} \qquad (59)$$

$$M = \frac{\alpha\,\overline{v}^2}{\overline{X}_1} = \frac{\alpha\,V_1^2}{\overline{X}_o} \qquad (60)$$

$$M' = \frac{6}{27.68g} \cdot \frac{w\,M}{a\,\bar{\omega}} = [7.82867 - 10]\frac{w\,M}{a\,\bar{\omega}} \qquad (61)$$

$$N = \frac{\lambda}{\overline{X}_o} \cdot \quad N' = \frac{\mu}{\overline{X}_o^2} = \frac{\mu\,N^2}{\lambda^2} \qquad (62)$$

$$P' = \frac{2}{27.68\,g}\,\frac{w\,V_1^2}{a\,\bar{\omega}} = [7.35155 - 10]\frac{w\,V_1^2}{a\,\bar{\omega}} \qquad (63)$$

$$f = \frac{1}{144 \times 6\,g} \cdot \frac{w\,V_1^2}{\bar{\omega}} = [5.55617 - 10]\frac{w\,V_1^2}{\bar{\omega}} \qquad (64)$$

$$\frac{a\,P'}{f} = \frac{1728}{27.68} \qquad (65)$$

$$\tau = \left(\frac{2 \times 27.68}{9\,\pi^2\,g\,p_o}\right)^{\frac{1}{2}} \frac{\overline{X}_o\sqrt{a\,w\,\bar{\omega}}}{d^2} = [8.56006 - 10]\frac{\overline{X}_o\,\sqrt{a\,w\,\bar{\omega}}}{d^2} \qquad (66)$$

$$v_c = \frac{l_o}{\tau} = [1.43994\,]\frac{l_o\,d^2}{\overline{X}_o\sqrt{a\,w\,\bar{\omega}}} \text{ (inches per second)} \qquad (67)$$

$$v_c' = v_c\left(\frac{p}{p_o}\right)^{\frac{1}{2}} = [9.41639 - 10]\ v_c\sqrt{p} \qquad (68)$$

$$\overline{X}_o = \frac{[1.43994]\,\tau\,d^2}{\sqrt{a\,w\,\bar{\omega}}} = \frac{[1.43994]\,l_o\,d^2}{v_c\sqrt{a\,w\,\bar{\omega}}} \qquad (69)$$

$$k = \frac{y}{\bar{\omega}} = \frac{v^2}{V_1^2\,X_2} \qquad (70)$$

It must be remembered that v and p refer to the period when the powder is burning and V and P to the period after the powder is all burned.

FRENCH UNITS

In metric units we shall take V_c in cubic decimetres, p in kilogrammes per square centimetre, d in centimetres and z_o, u and v in metres. Also $g = 9.80896$ m.s. With these units our formulas become,

$$\Delta = \frac{\bar{\omega}}{V_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

$$z_o = \frac{40}{\pi} \cdot \frac{a\,\bar{\omega}}{d^2} = [1.10491]\frac{a\,\bar{\omega}}{d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

$$V_1^2 = [2.76977]\frac{f\,\bar{\omega}}{w} \ (f \text{ in kilos. per cm.}^2) \quad . \quad . \quad . \quad (73)$$

$$M' = [7.70735 - 10]\frac{w\,M}{a\,\bar{\omega}} \ (\text{kilos. per cm.}^2) \quad . \quad . \quad (74)$$

$$P' = [7.23023 - 10]\frac{w\,V_1^2}{a\,\bar{\omega}} \ (\text{kilos. per cm.}^2) \quad . \quad . \quad (75)$$

$$f = [7.23023 - 10]\frac{w\,V_1^2}{\bar{\omega}} \ (\text{kilos. per cm.}^2) \quad . \quad . \quad (76)$$

$$v_c = [0.63128]\frac{l_o\,d^2}{\overline{X}_o\sqrt{a\,w\,\bar{\omega}}} \ (\text{cm. per. sec.}) \quad . \quad . \quad (77)$$

$$\overline{X}_o = [0.63128]\frac{l_o\,d^2}{v_c\sqrt{a\,w\,\bar{\omega}}} = [0.63128]\frac{\tau\,d^2}{\sqrt{a\,w\,\bar{\omega}}} \quad . \quad . \quad (78)$$

$$\tau = [9.36872 - 10]\frac{\overline{X}_o\sqrt{a\,w\,\bar{\omega}}}{d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

Characteristics of a Powder.—The quantities f, τ, α, λ and μ were called by Sarrau the characteristics of the powder because they determine its physical qualities. Of these quantities f depends principally upon the composition of the powder, and, with the same gun, for service charges, is practically constant for all powders having the same temperature of combustion. The value of τ depends generally upon the density and least

dimension of the grain. The factors α, λ and μ called "form characteristics" depend upon the form of the grain, and for the carefully moulded powders now employed their values may be determined with great precision. They are constant so long as the grain in burning retains its original form.

Expressions for M, M′, N and N′ in Terms of the Characteristics of the Powder.—When f and v_c are known from experimental firings or otherwise, for any gun and powder, the quantities V_1^2 and $\overline{X}_o$ can be determined either from (58) and (69), or (73) and (78). Substituting these in the proper expressions for M, M', N and N' they become

For English Units

$$M = [3.00389] \frac{\alpha\, f\, v_c}{d^2\, l_o}\left(\frac{a\, \bar{\omega}^3}{w}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (80)$$

$$M' = [0.83256] \frac{\alpha\, f\, v_c}{d^2\, l_o}\left(\frac{w\, \bar{\omega}}{a}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (81)$$

$$N = [8.56006 - 10] \frac{\lambda\, v_c}{d^2\, l_o} \sqrt{a\, w\, \bar{\omega}} \quad . \quad . \quad . \quad . \quad (82)$$

$$N' = \frac{\mu}{\lambda^2} N^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

For Metric Units

$$M = [2.48268] \frac{\alpha\, f\, v_c}{d^2\, l_o}\left(\frac{a\, \bar{\omega}^3}{w}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (84)$$

$$M' = [0.19003] \frac{\alpha\, f\, v_c}{d^2\, l_o}\left(\frac{w\, \bar{\omega}}{a}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (85)$$

$$N = [9.71291 - 10] \sqrt{a\, w\, \bar{\omega}} \quad . \quad . \quad . \quad . \quad (86)$$

$$N' = \frac{\mu}{\lambda^2} N^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

If we substitute the value of M' from (81) in (50) or (51), and reject the terms within the brackets, we have in effect Sarrau's

monomial formula for maximum pressure. But it is evident there can be no monomial formula for velocity or pressure unless λ and μ are approximately zero. Equations (80) to (87) are useful for determining the values of M and N (upon which all the other constants depend), when the charge varies or when there are variations in the weight of the projectile. In these formulas α, λ, μ and l_o are independent of $\tilde{\omega}$ and d and are strictly grain constants. v_c is a powder constant, varying only with the composition and density of the powder. f is approximately constant for full service charges of the same kind of powder, in guns of all calibers. For example the magazine rifle, caliber 0.3 inches, and the 16-inch B. L. R. give approximately the same value to f when computed by equation (64) or (65). This factor, however, varies with the charge in the same gun, for it is evident that its effective value as measured by projectile energy must decrease with the charge. Indeed if the charge be sufficiently reduced it is obvious that f becomes zero since we have omitted from our formulas all consideration of the force necessary to start the projectile. The law of variation is not known; but we will assume provisionally that f varies with the charge according to the law expressed by the equation

$$f = f_o\left(\frac{\tilde{\omega}}{\tilde{\omega}_o}\right)^n \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

where $\tilde{\omega}_o$ is the service charge by means of which M and N were determined and f_o the corresponding value of f computed by (64) or (76). If the weight of the projectile also varies we will assume that f may be determined by the equation

$$f = f_o\left(\frac{\tilde{\omega}}{\tilde{\omega}_o}\right)^n \left(\frac{w}{w_o}\right)^{n'} \quad . \quad . \quad . \quad . \quad . \quad (89)$$

The exponents n and n' must be determined from experimental data. If we make

$$K = \frac{f_o}{\tilde{\omega}^n\, w^{n'}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

(89) becomes

$$f = K\,\bar{\omega}^{n}\;w^{n'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (90')$$

Substituting this expression for f in (80) and (84) gives, for English units:

$$M = [3.00389]\,K\,\frac{\alpha\,v_c}{d^2\,l_o}\left(\frac{a\;\bar{\omega}^{3+2n}}{w^{1-2n'}}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (91)$$

and for metric units:

$$M = [2.48268]\,K\,\frac{\alpha\,v_c}{d^2\,l_o}\left(\frac{a\;\bar{\omega}^{3+2n}}{w^{1-2n'}}\right)^{\frac{1}{2}} \quad . \quad . \quad (91')$$

In the applications of these equations f_o must be computed by (64) or (76) and v_c by (67) or (77).

CHAPTER V

APPLICATIONS

THE principal formulas deduced in Chapter IV are here reproduced for convenience of reference. They are the following:

(a) **Formulas which Apply Only While Powder is Burning.**—

$$v^2 = M X_1 \{1 + N X_o + N' X_o^2\} \quad . \quad . \quad . \quad (1)$$

$$p = M' X_3 \{1 + N X_4 + N' X_5\} \quad . \quad . \quad . \quad (2)$$

It will be observed that these formulas for velocity and pressure are identical in form, and that the constants within the brackets are common to both. Also that M' is a simple multiple of M. Moreover, from the manner of deriving p from v^2, the velocity and pressure deduced from these formulas correspond at every point so that one can be easily and exactly computed from the other without the necessity of laying down velocity curves in order to obtain the pressures.

(b) **Formulas which Apply Only After the Powder has All Been Burned.**—

$$V^2 = V_1^2 X_2 = \frac{M \overline{X}_o X_2}{\alpha} = \frac{\lambda M}{\alpha N} X_2 \quad . \quad . \quad . \quad (3)$$

$$P = \frac{P'}{(1+x)^{\frac{4}{3}}} = P' (1 - X_2)^4 \quad . \quad . \quad . \quad . \quad (4)$$

(c) **Formulas Which Apply at the Instant of Complete Combustion.**—

$$\overline{v}^2 = M \overline{X}_1 \{1 + N \overline{X}_o + N' \overline{X}_o^2\}, \text{ (from (1))}: . \quad . \quad (5)$$

and

$$\overline{v}^2 = \frac{M \overline{X}_1}{\alpha} \text{ (from (3))} \quad . \quad . \quad . \quad . \quad (5')$$

Equations (5) and (5′) give the same value to $\overline{v}^2$, since the

former equation reduces to the latter at the point $\bar{u}$. But (1) and (3) are not tangent at the point $\bar{u}$ unless the vanishing surface (S') of the grain is zero, as with cubes, spheres, solid cylinders, etc.

From (2) we have at the travel $\bar{u}$

$$\bar{p} = M'\bar{X}_3\{1 + N\,\bar{X}_4 + N'\,\bar{X}_5\} \quad . \quad . \quad . \quad (6)$$

and from (4)

$$\bar{p} = \frac{P'}{(1+\bar{x})^{\frac{4}{3}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6')$$

Equations (6) and (6′) give the same value to $\bar{p}$ for all grains whose vanishing surface is zero, as may be thus shown:

Substituting for M' in (6) its value from (59), Chapter IV, and giving to N and N' their values in terms of $\bar{X}_o$, we have

$$\bar{p} = 3\,P'\frac{\bar{X}_3}{\bar{X}_o}\alpha\left\{1 + \frac{\bar{X}_4}{\bar{X}_o}\lambda + \frac{\bar{X}_5}{\bar{X}_o^{\,2}}\mu\right\}$$

But $\dfrac{\bar{X}_4}{\bar{X}_o} = 1 + \bar{X}$ and $\dfrac{\bar{X}_5}{\bar{X}_o^{\,2}} = 1 + 2\,\bar{X}$.

$$\therefore\ \bar{p} = 3\,P'\frac{\bar{X}_3}{\bar{X}_o}\alpha\,\{1 + \lambda + \mu + (\lambda + 2\,\mu)\,\bar{X}\}$$

For all grains whose vanishing surface is zero we have the relations (Equs. (10) and (12), Chapter III.)

$$\alpha\,(1 + \lambda + \mu) = 1$$

and

$$\alpha\,(\lambda + 2\,\mu) = -1$$

$$\therefore\ \bar{p} = 3\,P'\frac{\bar{X}_3}{\bar{X}_o}(1 - \bar{X}),$$

which readily reduces to (6′). Therefore for all forms of grain whose vanishing surface is zero the pressure curves (2) and (4) are tangent at $\bar{u}$. This is not true for grains for which $S' > 0$.

For these the pressure at travel $\bar{u}$ given by (2) is greater than that given by (4), and this difference increases with S'.

Monomial Formulas for Velocity and Pressure While the Powder is Burning.—The expressions for velocity and pressure while the powder is burning (equations (1) and (2)) are generally trinomials because equations (9) and (22), Chapter III, are trinomials. And these are so because of the geometrical characteristics α, λ and μ. In order to have a monomial expression for velocity or pressure λ and μ must both be zero. But upon this supposition (22), Chapter III, would become

$$k = \frac{\alpha\, l}{l_o}$$

or, the fraction of grain burned would be directly proportional to the thickness of layer burned; which is impossible, since the grain burns on all sides. This same supposition would also make

$$l_o\, S_o = V_o$$

which is not true, at least for finite volumes.

It has been shown in Chapter III, that all grains which, under the parallel law of burning, retain their original form until wholly consumed and for which $S' > 0$, have one or the other of the following expressions for α, namely, $1 + x$ or $2 + x$, x being the ratio of the thickness of web to the length (or breadth) of the grain. Only grains for which $\alpha = 1 + x$ can give approximate monomial expressions for velocity and pressure, and this by making x so small that it may be omitted in comparison with unity, in which case α becomes practically unity and λ and μ zero. To this class belong thin, flat grains and long cylindrical grains with axial perforation.

When $\alpha = 1$ and λ and μ are zero, equations (1) and (2) become

$$v^2 = M\, X_1 \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$p = M'X_3 = [7.82867 - 10]\, M\frac{w}{a\,\bar{\omega}}\, X_3 \quad . \quad . \quad . \quad (8)$$

Also, by equation (3), since $\alpha = 1$, we have, after the powder is all burned,

$$M = \frac{V_1^2}{X_o} = \frac{V^2}{X_o\, X_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

It will be seen that the pressures by the monomial formula are directly proportional to X_3, which therefore gives, to the proper scale, the typical pressure curve. Its maximum value, as seen from the table of the X functions, occurs when $x = 0.64$, and its logarithm is $9.86390 - 10$.* Applying this in equation (8) gives for the maximum pressure,

$$p_m = [7.69257 - 10]\, M\frac{w}{a\,\bar{\omega}} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If the maximum pressure, assumed to be the crusher-gauge pressure, is known by experiment, we may compute M from the last equation. Thus we have

$$M = [2.30743]\,\frac{a\,\bar{\omega}\,p_m}{w} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Substituting this in (7) and (8) we have while the powder is burning,

$$v = [1.15371]\left(\frac{a\,\bar{\omega}\,p_m}{w}\right)^{\frac{1}{2}} \sqrt{X_1} \quad . \quad . \quad . \quad . \quad (12)$$

and

$$p = [0.13610]\, p_m\, X_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Since by (12) the velocity is proportional to $\sqrt{X_1}$, this function represents the typical velocity curve while the powder is

* The maximum value of X_3 occurs when $x = 0.6336+$. But the value of x given above is near enough for all practical purposes. It may be noted here that the curve of X_3 has a point of inflection when $x = 1.3891$.

burning. After the powder is all burned the monomial formulas

$$V = V_1\sqrt{X_2} = [2.22191]\left(\frac{f\,\tilde{\omega}\,X_2}{w}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad (14)$$

and

$$P = \frac{P'}{(1+x)^{\frac{4}{3}}} = \frac{M'\,\overline{X}_o}{3\alpha\,(1+x)^{\frac{4}{3}}} \quad . \quad . \quad . \quad . \quad (15)$$

are to be employed.

Example.—As an example of monomial formulas for velocity and pressure take the following data from "Notes on the Construction of Ordnance," No. 89, pages 43–47:

Gun: 8-inch B. L. R., Model 1888. $V_c = 3617$ in.^{3}; $u_m = 205.25$ in.

Powder: Nitrocellulose composition, single-perforated grains of the following dimensions: length (m) 47.69 in.; outside diameter (2 R) 0.4455 in.; diameter of perforation (2 r) 0.1527 in.

From these dimensions we find by the formulas of Chapter III.,

$$2\,l_o = \frac{1}{2}(0.4455 - 0.1527) = 0.1464 \text{ in.}$$

$$x = \frac{2\,l_o}{m} = 0.0030525.$$

$$\alpha = 1 + x = 1.0030525.$$

$$\lambda = \frac{x}{1+x} = 0.0030433.$$

$$\mu = 0.$$

We may, therefore, in this case, assume $\alpha = 1$ and $\lambda = 0$ without material error, and employ the monomial formulas (7) and (8), computing M either by (9) or (11), according as we take the observed muzzle velocity or crusher-gauge pressure for this purpose. If the crusher-gauge pressure (assumed to be p_m) is employed equations (12) and (13) may be used. If it is known that the powder is all burned at, or near, the muzzle (9) becomes

$$M = \frac{v_m^2}{X_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9')$$

in which both symbols in the second member refer to the muzzle. If the charge is not all consumed at the muzzle and we know the value of v_c for the powder used, $\overline{X}_o$ can be found by (69), Chapter IV., and then N can be computed by (9). Finally if v_c is not known equations (12) and (13) must be employed.

As an example one shot was fired with a charge of 78 lbs., and a projectile weighing 318 lbs. The observed muzzle velocity was 2040 f. s., and crusher-gauge pressure 30450 lbs. per in.^{2}, and it was known that the combustion of the charge was practically complete at the muzzle. From the given data we find (taking $\delta = 1.567$), $\Delta = 0.5969$, $\log a = 0.01584$, and $z_o = 44.548$ in. Therefore,

$$x_m = \frac{u_m}{z_o} = \frac{205.25}{44.548} = 4.6073;$$

and from Table 1, for this value of x_m,

$$\log X_o = 0.77147$$
$$\log X_1 = 0.41207.$$
$$\log X_2 = 9.64060 - 10.$$
$$\therefore \log M = 2 \log 2040 - 0.41207 = 6.20719.$$

Also by (61), Chapter IV, $\log M' = 4.63036$.

The equations for the velocity and pressure curves for this shot are, therefore,

$$v = [3.10359] \sqrt{X_1} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and

$$p = [4.63036]\, X_3 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The first of these equations gives, of course, the observed muzzle velocity; and the second gives (by taking $x = 0.64$) a maximum pressure of 31208 lbs. per in.^{2}, exceeding the crusher-gauge pressure by 758 lbs.

If we determine the value of M by means of the crusher-gauge pressure we shall have by (11), $\log M = 6.19652$; and the equations for velocity and pressure now become

$$v = [3.09826] \sqrt{X_1}$$

and

$$p = [4.61969]\, X_3$$

This last equation gives the observed crusher-gauge pressure while the first makes the muzzle velocity 25 f. s. less than the observed. As muzzle velocities can be more accurately measured than maximum pressures, the first set of formulas are probably the more accurate and will be used in what follows in preference to the other set.

The expression for fraction of charge burned at any travel of projectile is found from (70), Chapter IV., and is for this example,

$$k = [3.02134 - 10]\, \frac{v^2}{X_2}. \quad . \quad . \quad . \quad . \quad (18)$$

The travel of projectile is given by the equation

$$u = z_o\, x = 44.548\, x \text{ inches} \quad . \quad . \quad . \quad . \quad (19)$$

The following table computed by means of equations (16), (17), (18), and (19), is represented by the curves v and p in Fig. 1.

x	Travel u inches	Velocity v ft. secs.	Pressure p lbs. per in.^{2}	Fraction of charge burned. k	Pressure P lbs. per in.^{2}	Velocity V ft.-secs.
0.0	0.0	0.0	00	0.0	84084	0.0
0.2	8.910	379.7	26115	0.257	65939	749.3
0.4	17.819	600.2	30260	0.357	53687	1005.1
0.6	26.729	770.3	31194	0.430	44931	1175.0
0.8	35.638	910.1	30949	0.489	38402	1301.6
1.0	44.548	1029.2	30214	0.539	33369	1401.5
1.2	53.457	1133.1	29281	0.583	29387	1483.4
1.4	62.367	1225.2	28285	0.623	26167	1552.3
1.6	71.277	1308.0	27288	0.659	23519	1611.5
1.8	80.186	1383.2	26322	0.692	21306	1663.1
2.0	89.096	1452.0	25401	0.722	19434	1708.6
2.5	111.370	1602.2	23322	0.790	15823	1802.8
3.0	133.644	1729.3	21548	0.849	13242	1877.0
3.5	155.918	1839.1	20034	0.901	11318	1937.5
4.0	178.192	1936.0	18733	0.948	9834	1988.2
4.5	200.466	2022.5	17606	0.991	8661	2031.5
4.6073	205.250	2040.0	17384	1.000	8441	2040.0

The last two columns in the table represented by the curves V and P, Fig. 1, show the velocity and pressure upon the supposition that the powder was all converted into gas at the tempera-

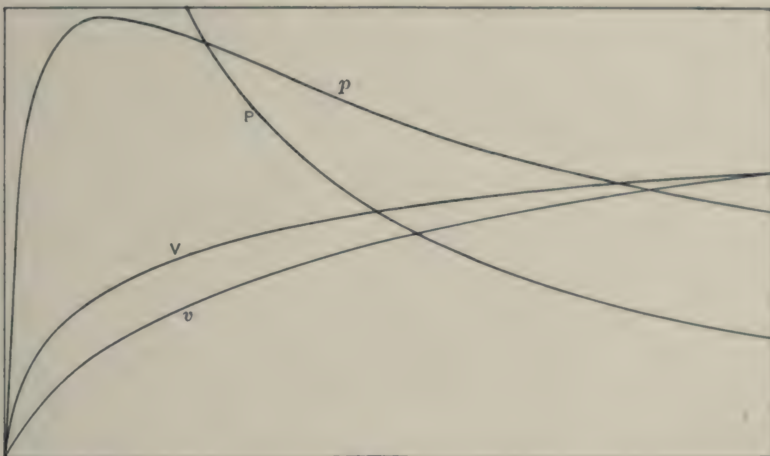


FIG 1.

ture of combustion before the projectile had moved. They were computed by the formulas

$$V^2 = V_1^{\,2} X_2 = [6.97866]\, X_2 \quad . \quad . \quad . \quad . \quad (20)$$

and

$$P = \frac{[4.92471]}{(1 + x)^{\frac{4}{3}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The force of the powder (f) and the velocity of combustion in free air (v_c) for this particular charge and brand of powder can now be computed by equations (64) and (67), Chapter IV. We find $f = 1396.9$ lbs. per in.^{2} and $v_c = 0.13614$ in. per sec.

If we wish to compute velocities and pressures in this 8-inch gun when the charge varies K must be computed by (90) and M by (91), Chapter IV. Since the weight of projectile is constant n' is zero; and for an 8-inch gun we will assume that $n = \frac{2}{3}$,

—this assumption to be tested by experiment. With these values of n and n' we have

$$K = \frac{1396.9}{(78)^{\frac{2}{3}}} = 76.519$$

and therefore,

$$f = 76.519\ \tilde{\omega}^{\frac{2}{3}}. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Substituting this value of K, and the gun and powder constants in (91), we have

$$M = [2.09974]\ a^{\frac{1}{2}}\ \tilde{\omega}^{\frac{13}{6}}. \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Also, from (61), Chapter IV,

$$M' = [2.43084]\ \frac{\tilde{\omega}^{\frac{7}{6}}}{a^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Therefore from (7), (8), and (10),

$$v = [1.04987]\ a^{\frac{1}{4}}\ \tilde{\omega}^{\frac{13}{12}} \sqrt{X_1} \quad . \quad . \quad . \quad . \quad (25)$$

$$p = [2.43084]\ \frac{\tilde{\omega}^{\frac{7}{6}}\ X_3}{a^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and

$$p_m = [2.29474]\ \frac{\tilde{\omega}^{\frac{7}{6}}}{a^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

which are the formulas for velocity and pressure for this gun and brand of powder in terms of the weight of charge.

As an example, what would be the maximum pressure with a charge of 79½ lbs.? We first find $\Delta = 0.6084$ and then $\log a = 0.00238$. We then have by (27)

$$\log p_m = 2.29474 + \frac{7}{6} \log 79.5 - \frac{1}{2} \log a = 4.51065$$

$$\therefore p_m = 32408 \text{ lbs. per in.}^2$$

This agrees very closely with observation.

We have the means of testing the accuracy of these equations to a limited extent, since there were four shots fired with charges of 70, 78, 85, and 88 lbs. The following table gives the results of all the necessary preliminary calculations for the four shots

fired and also for two others "estimated from prolonged empirical curves." The data from the shot fired with a charge of 78 lbs. have been taken as the basis of the calculations. The gun constants will be found on page 102.

$\bar{\omega}$ lbs.	Δ	log a	log z_0	At Muzzle			
				x	log X_0	log X_1	log X_2
60	0.4592	0.18742	1.70647	4.0347	0.74978	0.36944	9.61965−10
70	0.5357	0.08940	1.67540	4.3339	0.76150	0.39260	9.63110
78	0.5969	0.01584	1.64883	4.6073	0.77147	0.41207	9.64060
85	0.6505	9.95382	1.62414	4.8769	0.78066	0.42987	9.64919
88	0.6734	9.92777	1.61315	5.0018	0.78475	0.43770	9.65295
95	0.7370	9.86767	1.58629	5.3210	0.79467	0.45663	9.66196

The computed muzzle velocities and maximum pressures in the following table were obtained (witn the exception of the first two muzzle velocities) by equations (25) and (27). The values of f were computed by (22) and $\bar{X}_0$ by (69), Chapter IV.

$\bar{\omega}$ lbs.	log $\bar{X}_0$	f lbs. per inch²	Muzzle Velocity, f. s.			Maximum Pressures, Lbs. per In.²		
			Observed	Computed	O.−C.	Observed	Computed	O.−C.
60	0.74265	1173	1600	1600	0	18000	18859	−859
70	0.75819	1300	1839	1844	− 5	24889	25272	−383
78	0.77147	1397	2040	2040	0	30450	31206	−756
85	0.78382	1479	2200	2205	− 5	35600	37051	−1451
88	0.78931	1514	2275	2276	− 1	39301	39756	−455
95	0.80274	1593	2454	2441	+13	47280	46583	+697

For the first two shots the powder was all burned before the projectile had reached the muzzle, as is shown by the values of log $\bar{X}_0$. For these the muzzle velocities were computed by equation (14).

It will be observed that the equations by means of which the muzzle velocities and maximum pressures given in this table were computed depend for their constants upon one

measured velocity only, due to a charge of 78 lbs. The measured crusher-gauge pressure for this charge has not been made use of at all. The constant M upon which all the other constants depend might have been determined by equation (11) in which the muzzle velocity does not enter. But muzzle velocities can be more accurately measured than maximum pressures and are, therefore, better adapted to the determination of ballistic constants. The computed maximum pressures in the table are probably nearer the actual pressures on the base of the projectile than those given by the crusher gauge. The agreement between the computed and measured muzzle velocities is all that could be expected from any ballistic formulas.

To determine the travel of projectile when all the charge was burned we take $\bar{x}$ by interpolation from the table of the X functions corresponding to the values of $\log \bar{X}_o$. We then have:

$$\bar{u} = \bar{x}\, z_o.$$

For the travel of projectile when the pressure is a maximum. we have, calling this travel u',

$$u' = 0.64\, z_o.$$

The following table gives the values of u' and $\bar{u}$ for all the shots:

$\hat{\omega}$ lbs.	u' inches	$\bar{u}$ inches	$u_m - \bar{u}$ inches	k	Remarks
60	32.56	196.56	8.69	......	
70	30.31	201.13	4.12	......	
78	28.51	205.25	0.00	1.0000	
85	26.94	209.30	− 4.05	0.9926	
88	26.26	211.16	− 5.91	0.9895	
95	24.69	215.88	−10.63	0.9813	

It will be observed that as the charge increases the sooner it exerts its maximum pressure. The last column gives the fraction of the charge burned at the muzzle and shows that

approximately the entire charge for the series was consumed at the muzzle. k was computed by the formula

$$k = [6.17483-10]\frac{v^2}{\bar{\omega}^{\frac{1}{3}} X_2}$$

In order to determine the velocity and pressure curves for any given charge we should compute M and M' by equations (23) and (24), and then employ (7) and (8) as has already been done for a charge of 78 lbs. For example, determine the velocity and pressure curves for a charge of 95 lbs. We have, from (23) and (24),

$$\log M = 2.09974 + \frac{1}{2}\log a + \frac{13}{6}\log \bar{\omega} = 6.31864$$

$$\log M' = 2.43084 + \frac{7}{6}\log \bar{\omega} - \frac{1}{2}\log a = 4.80435$$

Therefore

$$v = [3.15932]\sqrt{X_1}$$

and

$$p = [4.80435]\,X_3$$

are the equations required.

Example.—Suppose the thickness of web of the grain we have been considering to be increased 10 per cent., all other conditions remaining the same. Deduce the velocity and pressure curves for a charge of 78 lbs. In this case it is evident that all the charge would not be burned in the gun and that in consequence both the maximum pressure and muzzle velocity would be diminished.

It will be seen from (69), Chapter IV, that, other things being equal, the value of $\bar{X}_o$ varies directly with the web thickness. Therefore if this is increased by 10 per cent., or, what is the same thing, is multiplied by 1.1, $\bar{X}_o$ will also be multiplied by 1.1; and from (60) and (61), Chapter IV, M and M' will be divided by 1.1. Therefore (16) and (17) will in this case become,

$$v = [3.08290]\sqrt{X_1},$$

and

$$p = [4.58897]\, X_3.$$

These equations give $v_m = 1945$ f. s., and $p_m = 28371$ lbs. This is a loss of 95 f. s. in muzzle velocity and a diminution of 2079 lbs. in maximum pressure. To determine the fraction of the charge burned at the muzzle, we have from (45), Chapter IV,

$$k = \frac{v^2}{V_1^2 X_2}\,;$$

which gives, by employing the muzzle velocity just computed,

$$k = 0.909.$$

Therefore on account of the increased thickness of web, seven pounds of the charge remained unburned when the projectile left the gun.

We may next inquire what effect a decrease of 10 per cent. in web thickness would have upon the muzzle velocity and maximum pressure. In this case we must multiply the original value of $\overline{X}_o$ by 0.9 and divide M and M' by the same fraction. We thus get

$$\log \overline{X}_o = 0.72571$$
$$\log M = 6.25295$$
$$\log M' = 4.67612$$

Therefore, from (9), the muzzle velocity in this case is found to be 2040 f. s.; and, by (10), the maximum pressure, 34675 lbs. That is, the muzzle velocity remains the same while the maximum pressure is increased by 4225 lbs. per in.^{2} These examples show that for the greatest efficiency (muzzle velocity and maximum pressure both considered), the web thickness for this form of grain should be such that the charge is all consumed at the muzzle. From the value of $\overline{X}_o$ given above we find, by interpolation, that $\overline{x} = 3.4890$; and, therefore, $\overline{u} = 155.43$ inches. For this travel the above values of M and M' give $\overline{v} = 1936$ f. s., and $\overline{p} = 22294$ lbs. The muzzle pressure comes out 8433 lbs.

Suppose for a hypothetical 7-inch gun we assume the following data:

$$d = 0''.7$$
$$V_c = 4{,}000 \text{ c. i.}$$
$$u_m = 40 \text{ calibers} = 280 \text{ inches.}$$
$$\Delta = 0.6.$$
$$\delta = 1.5776$$
$$f = 1396.9 \text{ lbs.}$$
$$v_c = 0.13614 \text{ in. per sec.}$$
$$w = 205 \text{ lbs.}$$

What muzzle velocity and maximum pressure would be obtained, supposing the charge to be all consumed at the muzzle; and what must be the thickness of web?

The weight of charge due to the given chamber capacity and density of loading is found to be 86.7 lbs. We next compute the following numbers by formulas given in Chapter IV:

$$\log a = 0.01221$$
$$\log z_o = 1.80713$$
$$x_m = 4.3655$$
$$\log \overline{X}_o = \log X_{om} = 0.76269$$
$$\log \overline{X}_1 = \log X_{1m} = 0.39492$$
$$\log V_1^2 = 7.21529 \text{ (By (58), Chapter IV),}$$
$$\log M = 6.45260 \quad \text{(By (9))}$$

Then by (7) and (10) we find

$$\text{Muzzle velocity} = 2653 \text{ f. s.}$$

and

$$\text{Maximum pressure} = 32112 \text{ lbs. per in.}^2$$

The muzzle pressure, by (8), is 18413 lbs.

The necessary thickness of web in order that the charge may all be consumed at the muzzle, is 0.158 inches. The other dimensions of the grains are immaterial.

If the volume of the chamber is taken at 3,000 c. i., all the

other data remaining the same, we should have the following results:

$$\tilde{\omega} = 65.03 \text{ lbs.}$$
$$M.V. = 2413 \text{ f. s.}$$
$$p_m = 28868 \text{ lbs. per in.}^2$$
$$M.P. = 14105 \text{ lbs. per in.}^2$$
$$2\, l_o = 0.152 \text{ in.}$$

If $V_c = 4500$ c. i., we have the following:

$$\tilde{\omega} = 97.54 \text{ lbs.}$$
$$M.V. = 2753 \text{ f. s.}$$
$$p_m = 33574 \text{ lbs. per in.}^2$$
$$M.P. = 24496 \text{ lbs. per in.}^2$$
$$2\, l_o = 0.160 \text{ in.}$$

Binomial Formulas for Velocity and Pressure.—Binomial formulas pertain to grains for which μ is zero or so small that it may be neglected, while λ must be retained on account of its magnitude. To this class belong all unperforated, long, slender grains of whatever cross-section, such as strips, ribbons, cylinders, etc. The binomial expressions for velocity and pressure for these grains are

$$v^2 = M\, X_1 \left\{1 - N\, X_o\right\} \quad . \quad . \quad . \quad . \quad (28)$$

and

$$p = M'\, X_3 \left\{1 - N\, X_4\right\} \quad . \quad . \quad . \quad . \quad (29)$$

The second term within the brackets has the negative sign because λ is always negative for these forms of grain.

Methods for Determining the Constants M and N.—The constants M and N can be determined when the given experimental data are such that two independent equations can be formed involving M and N. These data may be either two measured velocities of the same shot at different positions in the bore; or a measured muzzle velocity and crusher-gauge pressure,—the latter taken as the maximum pressure. In

addition to these all the elements of loading, as well as the powder and gun constants, are supposed to be known.

First Case.—Let v_1 and v_2 be two measured velocities in the bore at the distances u_1 and u_2 from the origin, which is the base of the projectile in its firing position. From the gun and firing constants compute z_o by (56), Chapter IV, and then x_1 and x_2 corresponding to u_1 and u_2 by the equation

$$x = \frac{u}{z_o}$$

With these values of x_1 and x_2 as arguments, interpolate from the table of the X functions the corresponding values of $\log X_o$ and $\log X_1$, distinguishing them by accents. We then have the two independent equations

$$v_1^2 = M\, X_1' \,(1 - N\, X'_o)$$
$$v_2^2 = M\, X_1'' \,(1 - N\, X''_o)$$

from which M and N may easily be determined. For simplicity let

$$b = \left(\frac{v_2}{v_1}\right)^2 \cdot \frac{X'_1}{X''_1} \text{ and } b' = \frac{X'_o}{X''_o}\,.$$

We then have in a form well adapted to logarithmic computation

$$N = \frac{1 - b}{(1 - bb')\, X''_o} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

and

$$M = \frac{v_1^2}{X'_1(1 - N\, X'_o)} = \frac{v_2^2}{X''_1(1 - N\, X''_o} \quad . \quad . \quad (31)$$

These equations are equally adapted to English or French units.

Second Case.—When the powder is not all burned in the gun let v_m be the observed muzzle velocity and p_m the crusher-gauge pressure. We then have the two independent equations

$$v_m^2 = M\, X_1\, (1 - N\, X_o)$$

and ((50′), Chapter IV),

$$p_m = [9.85640 - 10]\, M' \,(1 - [0.48444]\, N) \quad . \quad . \quad (32)$$

Substituting for M' its value in terms of M ((61), Chapter IV), and making, for English units,

$$c = [7.68507 - 10] \frac{w\, v^2_m}{a\, \bar{\omega}\, p_m\, X_1}$$

we have

$$N = \frac{1 - c}{X_o - 3.051\, c} = \frac{1 - c}{X_o \left(\dfrac{1 - [0.48444]\, c}{X_o} \right)}, \quad . \quad . \quad (33)$$

and then M from (31). The X functions in these last two formulas refer to v_m. Any measured velocity within the bore before the powder is all burned may be used instead of v_m. For French units the logarithmitic multiplier in the expression for c is [7.56404 − 10].

Second Method.—If the powder is all burned before the projectile reaches the muzzle, we have from (3)

$$M = \frac{\alpha\, V^2_m\, N}{\lambda\, X_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where V_m is the muzzle velocity and X_2 corresponds to V_m. N must be determined either by a velocity v_1 measured in the bore before the powder is consumed, or by the crusher-gauge pressure assumed to be p_m. If by the former, we have from (28)

$$v_1^2 = M\, X_1' \,(1 - N\, X'_o).$$

Substituting M from (34) in this equation and solving for N we have

$$N = \frac{1}{2\, X'_o} \left\{ 1 - \left(1 - \frac{4\, \lambda\, X_2}{\alpha\, X'_2} \left(\frac{v_1}{V_m} \right)^2 \right)^{\frac{1}{2}} \right\} \quad . \quad . \quad (35)$$

In this equation X'_o and X'_2 correspond to the measured velocity v_1. The travel of projectile to the point where all the powder is burned is found by the equation $\overline{X}_o = \lambda / N$ and a reference to the table of the X functions. In using these last

two formulas N must first be computed, and then M. Equation (35) is independent of the units employed.

If, as is usually the case, there is no interior measured velocity available recourse must be had to the crusher-gauge pressure p_m. In this case we have by means of (32) and (34), and (61), Chapter IV, for English units,

$$N = [9.21453 - 10]\left\{1 - \left(1 - [2.79937]\frac{4\lambda a \bar{\omega} X_2 p_m}{\alpha w V^2_m}\right)^{\frac{1}{2}}\right\} \quad (36)$$

and then M by (34). For metric units the logarithmetic multiplier within the braces becomes [2.92040].

Application to Sir Andrew Noble's Experiments.—These very important experiments were made at the Elswick works, Newcastle-on-Tyne, with a six-inch gun. They are thus described by Sir Andrew *: "The energies which the new explosives are capable of developing, and the high pressures at which the resulting gases are discharged from the muzzle of the gun, render length of bore of increased importance. With the object of ascertaining with more precision the advantages to be gained by length, the firm to which I belong has experimented with a six-inch gun of 100 calibers in length. In the particular experiments to which I refer, the velocity and energy generated has not only been measured at the muzzle, but the velocity and pressure producing this velocity have been obtained for every point of the bore, consequently the loss of velocity and energy due to any particular shortening of the bore can at once be deduced.

"These results have been attained by measuring the velocities every round at sixteen points in the bore and at the muzzle.

* "Report (1894) on methods of measuring pressures in the bore of guns"; and "Researches on Explosives, Preliminary Note." An abstract of these papers is given in the "English Text-book of Gunnery," 1902; in *Nature* for May 24, 1900; and in Encyclopædia Britannica, 11th edition, article "Ballistics."

These data enable a velocity curve to be laid down, while from this curve the corresponding pressure curve can be calculated. The maximum chamber pressure obtained by these means is corroborated by simultaneous observations taken with crusher gauges, and the internal ballistics of various explosives have thus been completely determined."

The velocities at the sixteen points in the bore were determined by registering the times at which the projectile passed these points. The registering apparatus is thus described by Sir Andrew in the "Report," page 11: "The chronograph which I have designed for this purpose consists of a series of thin disks made to rotate at a very high and uniform velocity through a train of geared wheels. The speed with which the circumference of the disks travels is between 1200 and 1300 inches per second, and, since by means of a vernier we are able to divide the inch into thousandths, the instrument is capable of recording the millionth part of a second.

"The precise rate of the disk's rotation is ascertained from one of the intermediate shafts, which, by means of a relay, registers the revolutions of a subsidiary chronoscope, on which, also by a relay, a chronometer registers seconds. The subsidiary chronoscope can be read to about the $\frac{1}{5000}$th part of a second.

"The registration of the passage of the shot across any of the fixed points in the bore is effected by the severance of the primary of an induction coil causing a spark from the secondary, which writes its record on prepared paper gummed to the periphery of the disk. The time is thus registered every round at sixteen points of the bore.

"I have ascertained by experiment that the mean instrumental error of this chronoscope, due chiefly to the deflection of the spark, amounts only to about three one-millionths of a second. Usually the pressures were deduced from the mean of three consecutive rounds fired under the same circumstances."

The following table gives the recorded experimental data for the various kinds of smokeless powders employed at the Elswick firings, and which will be used in the following discussions.

Kind of Powder	Weight of Charge, lbs.	Density of Loading	Mean Crusher-Gauge Pressure, lbs.	Measured Velocity when Projectile had Travelled			
				16.6 ft.	21.6 ft.	34.1 ft.	46.6 ft.
				f. s.	f. s.	f. s.	f. s.
Cordite, 0″.4....	27.5	0.55	47040	2794	2940	3166	3284
Cordite, 0″.35...	22.0	0.44	30352	2444	2583	2798	2915
Cordite, 0″.3....	20.0	0.40	36960	2495	2632	2821	2914
Ballistite, 0″.3...	20.0	0.40	33936	2416	2537	2713	2806

The cordite used in these experiments contained 37 per cent. of gun-cotton, 58 per cent. of nitro-glycerine, and 5 per cent. of a hydrocarbon known as vaseline. The ballistite was nearly exactly composed of 50 per cent. of dinitrocellulose (collodion cotton) and 50 per cent. of nitro-glycerine.

Discussion of the Data for Cordite, 0″.4 Diameter.—The form characteristics of cordite are $\alpha = 2$, $\lambda = -\frac{1}{2}$ and $\mu = 0$. The equations for velocity and pressure are therefore,

$$V^2 = M X_1 (1 - N X_o)$$

and

$$p = M' X_3 (1 - N X_4)$$

Since the second terms within the parentheses, which contain λ, have been made negative, λ in subsequent calculations must be regarded as positive.

For the preliminary calculations we have the following data:

$$\bar{\omega} = 27.5 \text{ lbs.}$$
$$w = 100 \text{ lbs.}$$
$$\Delta = 0.55$$
$$\delta = 1.56$$

From these data we find by the proper formulas:

$$\log a = 0.07084$$
$$\log z_o = 0.42178 \quad \therefore \quad z_o = 2.6411 \text{ ft.}$$

To determine whether the powder was all burned in the gun, the following table is formed which explains itself:

u ft.	$x = \frac{u}{z_0}$	v (observed) f. s.	$\log v^2$	$\log X_2$	$\log V_1^2$	$\log V_1$	V_1
16.6	6.2853	2794	6.89245	9.68496-10	7.20749	3.60374	4015
21.6	8.1784	2940	6.93669	9.71799	7.21870	3.60935	4068
34.1	12.9112	3166	7.00102	9.76657	7.23445	3.61722	4142
46.6	17.6446	3284	7.03281	9.79440	7.23841	3.61920	4161

The increase of V_1 as shown in the last column, indicates that the powder was all burned in the gun and between u = 34.1 and u = 46.6 ft.

We will compute M and N by means of the measured muzzle velocity (V_m) and the mean crusher-gauge pressure (p_m), as these data can always be obtained without sacrificing a gun.

For cordite equations (36) and (34) become

$$N = [9.21453 - 10] \left\{ 1 - \left(1 - [2.79973] \frac{a \, \tilde{\omega} \, X_2 \, p_m}{w \, V_m^2} \right)^{\frac{1}{2}} \right\} \quad (36')$$

and

$$M = \frac{4 \, V_m^2 \, N}{X_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (34')$$

The numbers to be used in these formulas are

$$\tilde{\omega} = 27.5 \text{ lbs.}$$
$$w = 100 \text{ lbs.}$$
$$p_m = 47040 \text{ lbs. per in.}^2$$
$$V_m = 3284 \text{ f. s.}$$
$$\log X_2 = 9.79440 - 10 \text{ (at muzzle)}$$
$$\log a = 0.07084$$

Performing the operations indicated in (36′) and (34′) we have

$$\log N = 8.73599 - 10$$
$$\log M = 6.57646$$

Also, by (62), Chapter IV,

$$\log M' = 4.89496.$$

The equations for the velocity and pressure curves while the powder is burning are, therefore,

$$v^2 = [6.57646]\, X_1 \{1 - [8.73599 - 10]\, X_o\} \quad . \quad (37)$$

and

$$p = [4.89496]\, X_3 \{1 - [8.73599 - 10]\, X_4\} \quad . \quad . \quad (38)$$

After the powder is burned we have from (34′), dropping the subscript from V_m, and reducing,

$$V = [3.61920]\sqrt{X_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

Also from (31) and (63), Chapter IV,

$$P = \frac{[5.07979]}{(1 + x)^{\frac{4}{3}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

To determine the travel of projectile to the point where the powder was all burned $\bar{u}$ we have, for cordite,

$$\bar{X}_o = \frac{1}{2N}$$

Therefore

$$\log \bar{X}_o = 0.96298;$$

and by interpolation from the table of the X functions,

$$\bar{x} = 16.018.$$

Whence

$$\bar{u} = \bar{x}\, z_o = 42.30 \text{ ft.}$$

The velocity $\bar{v}$ may be computed by either of equations (37) and (39), as they both give the same value to $\bar{v}$, namely,

$$\bar{v} = 3261 \text{ f. s.}$$

It will be seen that the increase of velocity from $\bar{u} = 42.26$ ft. to $u = 46.6$ ft., a travel of 4.34 ft., is only 23 f. s.

Since the vanishing surface of a grain of cordite is zero, (38) and (40) give the same value to $\bar{p}$. We find by either equation,

$$\bar{p} = 2745 \text{ lbs. per in.}^2$$

The muzzle pressure by (40) is 2431 lbs. The distance travelled by the projectile at point of maximum pressure is

$$0.45 \times 2.64 = 1.19 \text{ ft.} = 2.38 \text{ calibers.}$$

Equations (37) to (40) give all the information that was obtained by Noble's experiment with cordite, 0″.4. The only question that can arise is as to their accuracy in giving the velocity and pressure at every point of the bore. Equation (38) gives the observed maximum pressure and (37) the corresponding velocity. Equation (39) gives the observed muzzle velocity and (40) the corresponding pressure. These equations may be further tested by computing the velocities for 16.6, 21.6, and 34.1 feet travel and comparing with the measured velocities. The following table shows the results of this procedure. The differences between the measured and computed velocities are in all cases less than the probable error in measuring them, and are entirely negligible.

Travel of Projectile	Observed Velocity	Computed Velocity	O. – C.	Remarks
16.6 ft.	2794 f. s.	2781 f. s.	13	
21.6	2940	2942	−2	
34.1	3166	3172	−6	
46.6	3284	3284	0	

The limiting velocity, V_1, is 4161 f. s.

It only remains to compute the characteristics v_c and f to solve completely the problem pertaining to this round. These

are found to be

$$v_c = 0.38 \text{ inches per second,}$$

and

$$f = 2266 \text{ lbs. per in.}^2$$

This value of f would mean, if the problem under consideration were completely solved, that one pound of the gases of this powder, at temperature of combustion, confined in a volume of one cubic foot, would exert a pressure of 2,266 pounds per square inch. But the problem is very far from being solved rigorously. In the deduction of equation (18), Chapter II, which is the basis of all our formulas, there were neglected the following energies:

1. The heat lost by conduction to the walls of the gun.

2. The work expended on the charge, on the gun and carriage, and in giving rotation to the projectile.

3. The work expended in overcoming passive resistances, such as forcing, friction along the grooves, the resistance of the air, etc. In short the entire work of expansion was supposed to be employed in giving motion of translation to the projectile, and to be measured by the acceleration produced.

It may be seen, however, from a careful consideration of equation (18) and the use made of it in deducing the X functions that these functions are independent of the value of f; and that when this factor has been determined so as to satisfy completely such experiments as we have just been considering, these neglected energies are practically allowed for. Indeed, they are all contained implicitly in the factors M and N. Similar remarks apply to τ whose deduced value from (2), Chapter IV, depends upon the exponent of p_o/p, about which there is considerable uncertainty. But these characteristics are unnecessary for determining the equations of the velocity and pressure curves from such data as we have been considering. But they are of

use in deducing the circumstances of motion when the charge varies, as has been already shown.

We will now give a few illustrative examples which can be solved by this one round.

Example 1.—What thickness of layer was burned from the grains when the projectile had travelled 16.6 ft.?

Combining equations (12) and (14), Chapter IV, gives

$$l = \frac{l_o X_o}{\overline{X}_o}$$

That is, the thickness of layer burned from the surface of a grain of powder of whatever shape, in the bore of a gun, varies directly as the function X_o. In this example, applying the known values of l_o, $\overline{X}_o$ and X_o (for $u = 16.6$), we find

$$l = 0.1443 \text{ in.} \quad \text{Ans.}$$

Example 2.—What was the velocity of combustion of the grains at the point of maximum pressure?

We have (equation (53′), Chapter IV),

$$v'_c = v_c \left(\frac{p}{p_o}\right)^{\frac{1}{2}} = 0.3795 \left(\frac{47040}{14.6967}\right)^{\frac{1}{2}} = 21.47 \text{ in. per sec.} \quad \text{Ans.}$$

Example 3.—What must be the diameter of the grains of this powder in order that the charge of 27.5 lbs. should all be burned when the projectile has travelled 16.6 ft.?

When the only variation in the charge and conditions of loading is in the thickness of web, equation (53), Chapter IV, shows that the X_o function of the distance travelled by the projectile when the powder is all burned is directly proportional to the web thickness. We therefore find, by employing known numbers,

$$2\, l_o = 0.2885 \text{ in.} \quad \text{Ans.}$$

Cordite, 0″.35.—Preliminary calculations show that the cordite fired in this round was not quite all burned in the gun.

We will therefore compute N and M by (30) and (31) with the following data:

$$\log\ a = 0.21265$$
$$\log\ z_o = 0.46666 \quad \therefore\ z_o = 2.929 \text{ ft.}$$
$$v_1 = 2583 \text{ f. s.} = \text{velocity for 21.6 ft. travel.}$$
$$v_2 = 2915 \text{ f. s.} = \text{muzzle velocity.}$$
$$\log\ X'_o = 0.84614$$
$$\log X''_o = 0.96201$$
$$\log\ X'_1 = 0.55164.$$
$$\log X''_1 = 0.74763$$

These numbers give

$$\log\ N = 8.73582 - 10$$
$$\log M = 6.48164$$
$$\log M^1 = 4.75514$$

The results of further calculations for this round are:

$$p_m = 34093$$
$$\bar{u} = 46.96 \text{ ft.}$$
$$f = 2277 \text{ lbs.}$$
$$v_c = 0.315 \text{ in. per sec.}$$

The differences between the observed and computed velocities for $u = 16.6$ ft. and $u = 34.1$ ft., are, respectively, 12 and 6 f. s.

The limiting velocity for this round is 3731 f. s.

The mean crusher-gauge pressure was 30352 lbs., which is certainly erroneous. The force of the powder is practically the same as for cordite, 0″.4; but this latter seems to be a quicker powder than cordite 0″.35.

Cordite, 0″.3.—The cordite fired with this charge was all burned in the gun and we will, therefore, compute N and M by equations (35) and (34), with the following data:

$$\log\ a = 0.26928$$
$$\log\ z_o = 0.48190 \quad \therefore \quad z_o = 3.033 \text{ ft.}$$

$$v_1 = 2495 \text{ f. s.}$$
$$v_m = 2914 \text{ f. s.}$$
$$\log X_2 = 9.78256 - 10$$
$$\log X'_2 = 9.66596 - 10$$
$$\log X'_o = 0.79917$$

By means of these numbers the various formulas give

$$\log N = 8.80138 - 10$$
$$\log M = 6.54986$$
$$\log M' = 4.80822$$
$$\log P' = 4.92766$$
$$\log V_1^2 = 7.14642$$
$$\log \overline{X}_o = 0.89759$$
$$\overline{x} = 10.319$$
$$\overline{u} = 31.3 \text{ ft.}$$
$$\overline{v} = 2787.4 \text{ f. s.}$$
$$\overline{p} = 3331.2 \text{ lbs. per in.}^2$$
$$f = 2521 \text{ lbs.}$$
$$v_c = 0.309 \text{ in. per sec.}$$

The computed maximum pressure is 37287 lbs., which is but 327 lbs. in excess of the mean crusher-gauge pressure. The corresponding velocity is 877.2 f. s., and travel of projectile 1.33 ft. The differences between the observed and computed velocities for $u = 21.6$ ft., and $u = 34.1$ ft. are 1 and 5 f. s. respectively.

The limiting velocity is 3743 f. s.

This powder is apparently stronger than either the 0″.35 or 0″.4 cordite. These latter are evidently of the same composition, known as "Mark 1," while the former may have been the so-called "Cordite M. D.," which is said to have a slightly reduced rate of burning and to give higher velocities. Its composition is gun-cotton 65 per cent., nitro-glycerine 30 per cent., and mineral jelly 5 per cent.

Example.—Suppose the cordite, 0″.3, to be moulded into

cubes of the same web thickness. Determine the equations of velocity and pressure. We have $\alpha = 3$, $\lambda = -1$, and $\mu = \frac{1}{3}$, while V_1 and $\overline{X}_o$ remain the same as already found. We now have $M = \frac{3 V_1^2}{\overline{X}_o}$, $N = -\frac{1}{\overline{X}_o}$, and $N' = \frac{1}{3 \overline{X}_o^2}$: The equations are therefore,

$$v^2 = [6.72595]\, X_1 \{1 - [9.10241 - 10]\, X_o + [7.72770 - 10]\, X_o^2\}$$

and

$$p = [4.98431]\, X_3 \{1 - [9.10241 - 10]\, X_4 + [7.72770 - 10]\, X_5\}$$

The maximum pressure computed by this last formula is 45726 lbs. The muzzle velocity is, of course, the same as before, as is also the velocity $\bar{v}$.

Application to the Hotchkiss 57 mm. Rapid-Firing Gun.— The data for the following discussion are taken from a paper by Mr. Laurence V. Benét, printed in the *Journal U. S. Artillery*, Vol. 1, No. 3. The gun experimented with was a standard pattern, all steel, 57 mm. Hotchkiss rapid-firing gun, and the experiments consisted in "cutting off successive lengths from the chase and observing the velocities of a series of rounds fired with each resulting travel of projectile." The data necessary for this discussion are the following:

Gun Data.

Area of cross-section of bore, 0.2592 dm.^{2}
Equivalent diameter, 5.745 cm.
Net volume of powder chamber, 0.887 dm.^{3}

POWDER AND PROJECTILE.

"Two brands of the same type of smokeless powder were employed, both of which were manufactured at the Poudrerie Nationale de Sevran-Livry; they were designated as $B\,N_1$

and $B\ N_{144}$. These powders are in the form of thin strips, which are scored longitudinally on one side with a series of parallel and very narrow grooves. The chemical composition is unknown." The grains were of the following dimensions and densities:

	$B\ N_1$.	$B\ N_{144}$.
Length of strips,	76 mm.	85 mm.
Distance between scores,	1.4 mm.	1.6 mm.
Thickness of strips,	0.5 mm.	0.6 mm.
Specific gravity,	1.57	1.78

The elements of loading were as follows:

Weight of charge,	0.460 kilos.	0.400 kilos.
Weight of projectile,	2.720 kilos.	2.720 kilos.
Density of loading,	0.519	0.451

The velocities were measured by means of two Boulengé-Breger chronographs on independent circuits; and the pressures were determined by means of a crusher gauge seated in the breech block of the gun. The mean pressures at the breech were for $B\ N_1$ powder, 2547 kilos. per cm.^{2}; and for $B\ N_{144}$, 2543 kilos. per cm.^{2}

From the firing records was obtained the following table giving the velocity of the projectile corresponding to each length of travel in the bore:

Velocity in Bore with $B\ N_1$	Travel of Shell		Velocity in Bore with $B\ N_{144}$	Remarks
	Metres	Calibers		
Metres per Sec.			Metres per Sec.	
543.1	0.880	15.44	503.7	
574.4	1.051	18.44	534.9	
595.0	1.222	21.44	553.1	
612.6	1.393	24.44	565.0	
622.3	1.564	27.44	573.5	
636.5	1.792	31.44	591.0	
648.3	2.020	35.44	600.7	

We will first consider the powder $B\ N_1$, and compute by the

proper formulas—already many times referred to,—the values of a, z_o and x for the given charge and travels of projectile. Then take from the table of the X functions the logarithms of X_0, X_1 and X_2. All these are given in the following table for convenient reference:

$$\log a = 0.11053. \quad \log z_o = 9.35962 - 10.$$

u Metres	x	$\log X_0$	$\log X_1$	$\log X_2$	Remarks
0.880	3.8447	0.74183	0.35356	9.61173	
1.051	4.5918	0.77092	0.41100	9.64008	
1.222	5.3389	0.79521	0.45765	9.66244	
1.393	6.0860	0.81604	0.49670	9.68067	
1.564	6.8362	0.83425	0.53013	9.69590	
1.792	7.8293	0.85540	0.56819	9.71279	
2.020	8.8254	0.87382	0.60062	9.72681	

It is known that the powder was all burned in the gun, as might be also inferred from the thinness of web; and the first step is to determine the travel of projectile when this takes place, in other words the value of $\bar{u}$. On account of the "series of parallel and very narrow grooves" with which the strips were scored on one side, it is difficult to ascertain the form characteristics from geometrical considerations. Their determination will therefore be left until M and N are computed from the measured velocities. μ will be considered zero.

The expression for V_1, the limiting velocity, is

$$V_1 = \frac{V}{\sqrt{X_2}},$$

where V is any velocity after the powder is all burned and X_2 a function of the corresponding travel of projectile. If then we compute V_1 by this formula for all the measured velocities, and find that it is approximately constant for a certain number of measured velocities nearest the muzzle, we shall have an indication of the travel of projectile when the powder

is all burned. The following table gives the values of V_1 so computed:

u	V_1
0.880 m.	849 m. s.
1.051	869
1.222	878
1.393	885
1.564	883
1.792	886
2.020	888

An examination of this table shows that $\bar{u}$ lies between 1.222 and 1.393, or that $\bar{x}$ lies between the numbers 5.3389 and 6.0860 and rather nearer the former than the latter.

We will assume $\bar{x} = 5.6$ and $V_1 = 885.5$ m. s., which is a mean of the last four values. Since

$$\bar{v} = V_1 \sqrt{\bar{X}}$$

we find

$$\bar{v} = 605.1 \text{ m. s.}$$

N and M can now be computed by (30) and (31), which do not contain the form characteristics.

The data are: $v_1 = 543.1$ m. s., $v_2 = 605.1$ m. s., $\log X'_o = 0.74183$, $\log X''_o = 0.80284$, $\log X'_1 = 0.35356$, and $\log X''_1 = 0.47205$.

The results of the calculations are:

$$\left.\begin{aligned} \log N &= 8.68636 - 10 \\ \log M &= 5.25171 \\ \log M' &= 3.62063 \end{aligned}\right\} \text{While powder is burning.}$$

$$\left.\begin{aligned} V &= [2.94719] \sqrt{X_2} \\ P &= \frac{[3.78589}{(1 + x)^{\frac{4}{3}}} \end{aligned}\right\} \text{After powder is all burned.}$$

The following table shows the agreement between the observed and computed velocities:

Travel of Projectile	Velocities		O.−C.	Remarks
	Observed	Computed		
0.880 m.	543.1 m. s.	543.1 m. s.	0.0	
1.051	574.4	572.8	1.6	
1.222	595.0	597.4	−2.4	
1.393	612.6	613.1	−0.5	
1.564	622.3	623.9	−1.6	
1.792	636.5	636.3	0.2	
2.020	648.3	646.5	1.8	

The greatest of these differences is less than one-half of one per cent. of the observed velocity and the others are practically nil. The maximum pressure computed by the formula

$$p_m = [9.85640 - 10]\, M' \left\{1 - [0.48444]\, N\right\}$$

is 2555 kilos. per cm.², differing by less than one-third of one per cent. of the mean crusher-gauge pressure. These results show that the assumed value of $\overline{x} = 5.6$ is practically correct.

Finally, we have,

$$f = 7883 \text{ kg. per cm.}^2$$

and

$$v_c = 0.438 \text{ cm. per sec.}$$
$$= 0.172 \text{ in. " "}$$

The form characteristics α and λ can be computed by the formulas

$$\alpha = \frac{M\,\overline{X}_o}{V_1^2} \text{ and } \lambda = N\,\overline{X}_o$$

From these we find $\alpha = 1.4460$ and $\lambda = 0.3045$.

For the $B\,N_{144}$ powder the equations for the velocity and pressure curves are found, by a process entirely similar to the above, to be,—while the powder is burning,—

$$v^2 = [5.24187]\, X_1 \left(1 - [8.75166 - 10]\, X_o\right)$$

and

$$p = [3.56310]\, X_3 \left(1 - [8.75166 - 10]\, X_4\right)$$

After the powder is burned the equations become

$$V = [2.92007]\sqrt{X_2}$$

and

$$P = \frac{[3.68425]}{(1+x)^{\frac{4}{5}}}$$

The following table shows the agreement between the observed and computed velocities:

Travel of Projectile m.	Observed Velocity m. s.	Computed Velocities m. s.	O.−C.	Remarks
0.880	503.7	503.9	−0.2	
1.051	534.9	532.2	2.7	
1.222	553.1	553.6	−0.5	
1.393	565.0	566.0	−1.0	
1.564	573.5	576.5	−3.0	
1.792	591.0	588.4	2.6	
2.020	600.7	598.5	2.2	

The value of f for $B\ N_{144}$ comes out 8001.5 kilos. per cm.^{2}, and v_c is found to be 0.5268 cm. per sec. This powder is therefore slightly "stronger" than $B\ N_1$ and about 22 per cent. quicker; and this notwithstanding its greater density.

Application to the Magazine Rifle, Model of 1903.—The following data pertaining to this rifle were obtained partly from a descriptive pamphlet issued by the Ordnance Department, and partly through the courtesy of officers of the Ordnance Department on duty at the Springfield Armory and Frankford Arsenal, to whom the writer is under special obligations:

Caliber, 0.3 inches.

Volume of chamber, 0.252 cubic inches.

Total travel of bullet in bore, 22.073 inches.

Mean weight of powder charge, 44 grains.

Weight of bullet, 220 grains.

"The standard muzzle velocity of this ammunition is 2300

f. s., with an allowed mean variation of 15 f. s. on either side of the standard. The powder pressure in the chamber is about 49,000 pounds per square inch."

The powder used with this rifle is composed essentially of 70 per cent. nitrocellulose and 30 per cent. nitro-glycerine. "The grains are tubular, being formed by running the powder colloid through a die 0.09 inch in diameter, with a pin 0.03 inch in diameter; and the string thus made is cut 21 to the inch." There are considerable variations in the length and diameter of the grains "due to the fact that the string is not cut exactly perpendicular to its axis, and to irregularities in shrinking. There are 83,000 to 91,000 grains per pound. The specific gravity is about 1.65, and the gravimetric density is from 0.90 to 0.94."

On account of the tubular form of the grains the characteristic μ is zero, and therefore the equations for velocity and pressure are binomials. We have reliable measured interior velocities for this rifle, obtained at the Springfield Armory in the fall of 1903, by firing with a rifle the barrel of which was successively cut off one inch. Five shots (sometimes more) were fired for each length of barrel and the velocities were measured at a distance of 53 ft. from the muzzle, and reduced to muzzle velocity by well-known methods. (See Table A.)

It is known that the charge in the magazine rifle is all burned at, or very near, the muzzle. We may, therefore, take the two extreme reduced velocities of the series for v_1 and v_2 and thereby minimize the effects of errors in measuring the velocities. The firing data are then,

$$v_1 = 1274 \text{ f. s.}; \qquad v_2 = 2277.6 \text{ f. s.}$$
$$u_1 = 3.073 \text{ in.}; \qquad u_2 = 20.073 \text{ in.}$$

The weight of charge in these firings was 45.1 grains and weight of bullet 220 grains. The preliminary calculations give

$$\Delta = 0.7077$$
$$\log a = 9.90686 - 10$$
$$\log z_o = 0.30878 \qquad \therefore \quad z_o = 2.036 \text{ in.}$$
$$x_1 = 1.5093, \qquad \log X'_o = 0.57969, \quad \log X'_1 = 0.00146.$$
$$x_2 = 10.84125, \quad \log X''_o = 0.90504, \quad \log X''_1 = 0.65421.$$

These numbers and the velocities v_1 and v_2, substituted in (30) and (31), give

$$\log N = 8.73379 - 10$$

and

$$\log M = 6.30896.$$

We also find

$$\log M' = 4.91902.$$

The formulas for velocity and pressure are, therefore,

$$v^2 = [6.30896]\, X_1 \left\{1 - [8.73379 - 10]\, X_o\right\}$$
$$p = [4.91902]\, X_3 \left\{1 - [8.73379 - 10]\, X_4\right\}$$

We have

$$\alpha = \frac{M\,\overline{X}_1}{\overline{v}^2}$$

and this substituted in (15), Chapter IV, gives

$$y = \tilde{\omega}\,\frac{M\,\overline{X}_2}{\overline{v}_2}\, X_o \left\{1 - [8.73379 - 10]\, X_o\right\}$$

Since in this case $\overline{v}$ and $\overline{X}_2$ refer to the muzzle, we have for the powder burned, in grains,

$$y = [0.99736]\, X_o \left\{1 - [8.73379 - 10]\, X_o\right\}$$

or, in another form more convenient for computation,

$$y = [4.68840 - 10]\,\frac{v^2}{X_2}$$

Table A gives the measured and computed velocities for the travels of projectile in the first column. and also the weight of powder burned at each travel.

TABLE A

Travel of Projectile, inches	Mean Velocity 53 Feet from Muzzle, f. s.	Muzzle Velocity Deduced from Measured f. s.	Computed Velocity, f. s.	O. − C.	Powder Burned, grains
3.073	1253	1274	1274	0	29.99
4.073	1402	1426	1432	− 6	32.61
5.073	1531	1558	1555	3	34.63
6.073	1633	1662	1656	6	36.25
7.073	1742	1772	1740	32	37.59
8.073	1771	1802	1812	−10	38.71
9.073	1860	1894	1874	20	39.66
10.073	1909	1943	1929	14	40.48
11.073	1957	1992	1976	16	41.19
12.073	1989	2023	2018	5	41.81
13.073	2016	2052	2057	− 5	42.36
14.073	2050	2086	2091	− 5	42.83
15.073	2069	2105	2122	−17	43.25
16.073	2104	2140	2151	−11	43.63
17.073	2129	2165	2177	−12	43.95
18.073	2183	2219	2200	19	44.25
19.073	2163	2200	2222	−22	44.50
20.073	2201	2238	2242	− 4	44.73
21.073	2203	2240	2261	− 1	44.93
22.073	2240	2278	2278	0	45.10

Table B, on page 134, supplements Table A by giving computed velocities and pressures from the origin of motion. The velocity curve in the diagram, Fig. 2, on page 135, shows at a glance the agreement between theory and observation.

It will be observed that the computed pressures depend entirely upon two measured velocities. Also that the maximum pressure occurs when $x = 0.45$, and agrees with the official statement. The muzzle pressure is about 6,000 lbs. per in.^{2}

Powder Characteristics.—The form characteristics of these grains according to the given dimensions are

$$\alpha = 1.63 \text{ and } \lambda = 0.3865.$$

But these minute grains, of which there are 560 in the service charge, shrink irregularly and many of them doubtless are more or less abraded and perhaps broken, so that it is impossible to determine the mean values of α and λ geometrically with any

TABLE B

x	u, inches	Computed Velocity, f. s.	Computed Pressure, lbs. per inch²	Powder Burned, grains	Pressure on Base of Projectile, pounds
0.000	0.000	0.000	0.00000	0.000	000
0.001	0.002	8.590	4501	1.081	318
0.01	0.020	47.854	13835	3.375	978
0.1	0.204	254.91	37008	10.139	2556
0.2	0.407	408.98	45117	13.842	3189
0.3	0.611	531.70	48456	16.475	3425
0.4	0.814	635.14	49640	18.554	3509
0.45	0.916	681.48	49769	19.454	3518
0.5	1.018	724.89	49695	20.283	....
0.6	1.222	804.25	49118	21.766	....
0.8	1.629	939.77	47039	24.221	....
1.0	2.036	1052.5	44492	26.205	....
1.2	2.443	1148.8	41882	27.867	....
1.4	2.850	1232.6	39369	29.291	....
1.6	3.258	1306.4	37023	30.533	....
2.0	4.072	.....	32854	.....	....
2.5	5.090	.....	28550	.....	....
3	6.108	.....	25061	.....	....
4	8.144	.....	19812	.....	....
5	10.180	.....	16086	.....	....
6	12.216	.....	13319	.....	....
7	14.252	.....	11188	.....	....
8	16.288	.....	9501	.....	....
9	18.324	.....	8134	.....	....
10	20.360	.....	7006	.....	....
11	22.396	.....	6045	.....	....

certainty. They may, however, be deduced from the values of M and N. From these we find

$$\alpha = 1.7710$$

$$\lambda = 0.4353$$

Finally we find

$$f = 1622.5 \text{ lbs. per in.}^2$$

and

$$v_c = 0.28 \text{ in. per second.}$$

Formulas for Designing Guns for Cordite.—The caliber, of course, is given, and the weight of the projectile of desired length

and form of head can be computed by known methods.* The grain characteristics and density of cordite and the values of f

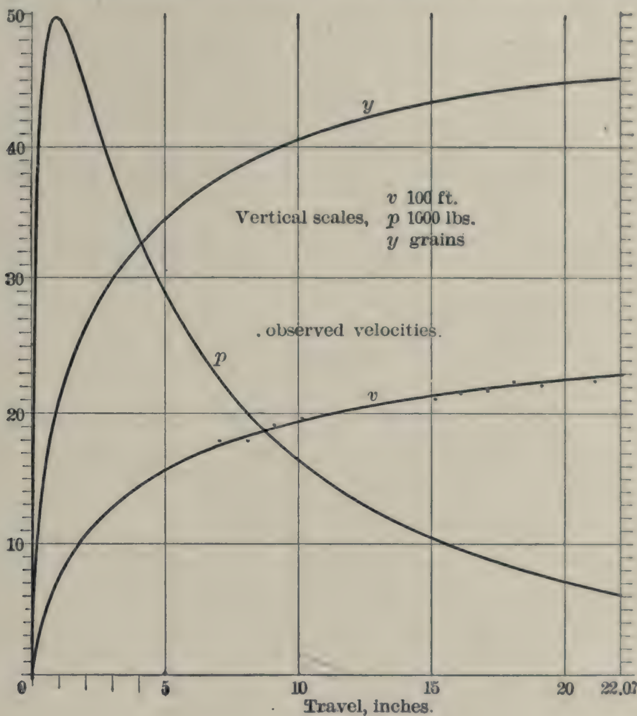


FIG. 2.

and v_c are also known. The necessary formulas for this discussion, given in the order in which they will be used, are the following:—

$$\tilde{\omega} = [8.55783]\ \Delta\ V_c \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$a = \frac{1}{\Delta} - \frac{1}{\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$z_o = [1.54708]\,\frac{a\,\tilde{\omega}}{d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

* See the author's "Handbook" (Artillery Circular N), chapter xi.

$$V_1^2 = [4.44383]\frac{f\,\tilde{\omega}}{w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

$$\overline{X}_o = b\left\{ 1 + \left(1 - \frac{[0.48444]}{b} \right)^{\frac{1}{2}} \right\} \quad . \quad . \quad . \quad (e)$$

in which

$$b = \frac{[2.12890]\,f}{a\,p_m}$$

This equation is deduced from (32), eliminating M' and N.

$$M = \frac{2\,V_1^2}{\overline{X}_o};\quad N = -\frac{1}{2\,\overline{X}_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

$$M' = [7.82867 - 10]\,M\,\frac{w}{a\,\tilde{\omega}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g)$$

$$v^2 = M\,X_1\,[1 - N\,X_o] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (h)$$

$$p = M'\,X_3\,(1 - N\,X_4) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

$$p_m = [9.85640 - 10]\,M'\,(1 - [0.48444]\,N) \quad . \quad . \quad (j)$$

$$l_o = [8.56006 - 10]\frac{v_c\sqrt{a\,w\,\tilde{\omega}}}{d^2}\,\overline{X}_o \quad . \quad . \quad . \quad . \quad (k)$$

$$k = \frac{v^2}{V_1^2\,X_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (l)$$

$$x = \frac{u}{z_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (m)$$

Example.—Take the hypothetical 7-inch gun already considered on page 111, for which $d = 7''$ and $w = 205$ lbs. For cordite of $0''.3$ diameter we found $f = 2521$ lbs. per in.², and $v_c = 0.309$ in. per sec. Also $\delta = 1.56$. The only assumptions necessary are the volume of the chamber (V_c) and the density of loading. And this last is not purely arbitrary, since considerations of safety to the gun and its efficiency restrict its value to narrow limits,—say from 0.4 to 0.6. This latter value is often exceeded, especially in our service; but it is believed that by choosing the proper shape and size of grain this can always be

avoided. As f and v_c are unusually large for cordite, we will take $\Delta = 0.4$; and for a first assumption will give the chamber a volume of 3,000 c. i., which is less than the volume of the chamber of the 6-inch wire-wound gun. Finally we will take $p_m = 37,000$ lbs. per in.², leaving the muzzle velocity and travel in the bore for later consideration.

From the given data we find, by means of the above formulas, $\bar{\omega} = 43.352$ lbs., $\log a = 0.26928$, $\log z_o = 1.76317$, $\log V_1^2 = 7.17066$, $\log \overline{X}_o = 0.90184$, $\log M = 6.56985$, $\log M' = 4.80398$ and $\log N = 8.79713 - 10$.

The equations for velocity and pressure are, therefore,

$$v^2 = [6.56985]\, X_1\, (1 - [8.79713 - 10]\, X_o)$$

and

$$p = [4.80398]\, X_3\, (1 - [8.79713 - 10]\, X_4)$$

This last equation, which is the same as equation (j) when $x = 0.45$, makes $p_m = 37000$, thus verifying the calculations.

The muzzle velocity will, of course, depend upon where we place the muzzle, in other words upon the value adopted for u_m. If we regard 40 calibers as a suitable travel in the bore, we shall have

$$u_m = 40 \times 7'' = 280''$$

whence

$$x_m = u_m / z_o = 4.8305$$

For this value of x, Table I gives

$$\log X_o = 0.77912$$
$$\log X_1 = 0.42689$$

and these in the above velocity equation give

$$v_m = 2487 \text{ f. s.}$$

We find, from Table I, taking $\log \overline{X}_o$ as the argument,

$$\bar{x} = 10.613;$$

and by (m)

$$\bar{u} = 615.19 \text{ in.}$$

Also by (k)

$$2\,l_o = 0''.47$$

and by (l)

$$k_m = 0.9394.$$

That is 94 per cent. of the charge was burned at the assumed muzzle.

If the maximum pressure is increased to 38,000 lbs., the density of loading and volume of chamber remaining as before, the velocity for a travel of 280 inches will be increased to 2503 f. s., and the thickness of web, or diameter of the grain, will be reduced to $0''.45$. This slight diminution in the diameter of the grain increases the initial surface of combustion of the charge about 3½ per cent., which fully accounts for the increased maximum pressure.

If we take $\Delta = 0.5$, $V_c = 3{,}000$ c. i. and $p_m = 37{,}000$ lbs. per in.^{2}, there results $\tilde{\omega} = 54.19$ lbs., $v_{280} = 2570$ f. s., and $2\,l_o = 0''.66$.

Trinomial Formulas for Velocity and Pressure.—Trinomial formulas occur when the grains of which the charge is composed are of such form and dimensions that the form characteristic μ cannot be regarded as zero. Spherical, cubical, and multi-perforated cylindrical grains are of this kind. For the first two forms mentioned the second term is negative and the third positive; while for m.p. grains (those used in our service), the second term is positive and the third negative.

For spherical and cubical grains we may have, before the powder is all burned, the two independent equations,

$$v_1^2 = M\,X_1'\left(1 - N\,X_o' + \frac{1}{3}N^2 X_o'^2\right)$$

$$v_2^2 = M\,X_1''\left(1 - N\,X''_o + \frac{1}{3}N^2 X''^2_o\right)$$

Put for convenience,

$$a = \left(\frac{v_2}{v_1}\right)^2 \frac{X'_1}{X_1''}, \; b = \frac{X'_o}{X''_o},$$

$$c = \frac{3\,(1 - a\,b)}{2\,(1 - a\,b^2)\,X''_o} \text{ and } d = \frac{2\,c\,(1 - a)}{(1 - a\,b)\,X''_o}.$$

Then the quadratic equations give, using the sign applicable to this problem,

$$N = c - \sqrt{c^2 - d} = c\left(1 - \left(1 - \frac{d}{c^2}\right)^{\frac{1}{2}}\right) \quad . \quad . \quad (41)$$

The value of M may now be computed by either of the above expressions for v^2. Or, if V_m is the muzzle velocity, that is, if the powder is all burned in the gun, M may be computed by the formula derived from (3),

$$M = \frac{3\,V_m{}^2\,N}{X_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

If the powder is not all burned in the gun and our data are a muzzle velocity and the crusher-gauge pressure (assumed to be the maximum pressure), N may be computed by the following process: Compute the auxiliary quantities b, c, and d by the formulas:

$$b = [7.68507 - 10]\,\frac{w\,v_m{}^2}{a\,\bar{\omega}\,p_m\,X_1}\,;\; c = \frac{3\left(1 - \dfrac{b\,X_4}{X_o}\right)}{2\,X_o\left(1 - \dfrac{b\,X_5}{X^2_o}\right)}$$

$$d = \frac{3\,(1 - b)}{X^2_o\left(1 - \dfrac{b\,X_5}{X^2_o}\right)}$$

Then

$$N = c\left\{1 - \left(1 - \frac{d}{c^2}\right)^{\frac{1}{2}}\right\} \quad . \quad . \quad . \quad . \quad . \quad (43)$$

The functions X_4 and X_5 pertain to the tabular value of x which gives the maximum pressure. If we take this to be 0.45

no material error will ensue. We therefore have

$$\log X_4 = 0.48444$$
$$\log X_5 = 0.93587$$

The function X_o pertains to the muzzle.

It should be remembered that equations (41), (42), and (43) are applicable to cubical and spherical grains only.

Application to Noble's Experiments with Ballistite.—The ballistite consisted of equal parts of dinitrocellulose and nitroglycerine and was in the form of cubes 0.3 of an inch on a side. The gun, powder, and firing data are as follows: $d = 6$ inches, $\Delta = 0.4$, $\delta = 1.56$, $\tilde{\omega} = 20$ lbs., and $w = 100$ lbs. From these we find $\log a = 0.26928$ and $\log z_o = 0.48190$. $\therefore z_o = 3.033$ ft. The following table, which explains itself, is formed for convenient reference:

u, ft.	$x=u/z_0$	Observed Velocity f. s.	$\log X_0$	$\log X_1$	$\log X_2$	Remarks
16.6	5.473	2416	0.79917	0.46513	9.66596-10	
21.6	7.121	2537	0.84069	0.54181	9.70112	
34.1	11.242	2713	0.91049	0.66339	9.75289	
46.6	15.363	2806	0.95685	0.73940	9.78256	

As the powder was not quite all burned in the gun the extreme measured velocities are available for determining N and M by means of (41). The data are $v_1 = 2416$ f. s., $v_2 = 2806$ f. s., $\log X'_o = 0.79917$, $\log X''_o = 0.95685$, $\log X'_1 = 0.46513$ and $\log X_1'' = 0.73940$. Substituting these in (41), we find

$$\log N = 9.03843 - 10$$

and then

$$\log N' = 7.59974 - 10$$
$$\log M = 6.62918$$
$$\log M' = 4.88754$$

The equations for velocity and pressure are, therefore,

$$v^2 = [6.62918]\, X_1 \left\{ 1 - [9.03843 - 10]\, X_o + [7.59974 - 10]\, X_o^2 \right\}$$
$$p = [4.88754]\, X_3 \left\{ 1 - [9.03843 - 10]\, X_4 + [7.59974 - 10]\, X_5 \right\}$$

The equation for velocity will, of course, give the observed velocities for $u = 16.6$ ft., and $u = 46.6$ ft. It should also give the observed velocities, if our method is correct, for $u = 21.6$ ft. and $u = 34.1$ ft., and indeed for every point in the bore from the firing seat to the muzzle. The velocities computed for these intermediate values of u are 2536.5 f. s., and 2710 f. s., respectively, which differ so slightly from the measured velocities as to be negligible. The maximum pressure, which occurs in this case when $x = 0.4$, is 39,163 lbs. per in.^{2}; and the corresponding velocity 865.8 f. s., and travel of projectile 1.213 ft.

The distance travelled by a projectile to the point where the powder is all burned is determined by means of $\overline{X}_o$ and the table of the X functions. We have for cubical grains for which λ is unity, $\overline{X}_o = 1/N$. Therefore for this example,

$$\log \overline{X}_o = 0.96157$$

Corresponding to this value of $\log \overline{X}_o$, we find, by interpolation from the table,

$$\bar{x} = 15.8649,\ \log \overline{X}_1 = 0.74694, \text{ and } \log \overline{X}_2 = 9.78536 - 10.$$

To compute $\bar{u}$, we have $\bar{u} = 15.8649 \times 3.033 = 48.12$ ft. The charge was therefore not all burned in the gun, though the fraction of the charge remaining unburned was exceedingly small, practically zero. To get an expression for the fraction of the charge burned for any travel of the projectile, we have, from (45), Chapter IV,

$$v^2 = k\, V_1^2\, X_2$$

But from (3), for cubic grains,

$$V_1^2 = \frac{M}{3\,N} \quad \therefore \log V_1^2 = 7.11363 \text{ (for this example).}$$

Therefore for cubic grains,

$$k = \frac{3\,N\,v^2}{M\,X_2},$$

which for this example reduces to

$$k = [2.88637 - 10]\frac{v^2}{X_2}$$

Applying different velocities and the corresponding values of $\log X_2$ in this formula it will be found that the charge was practically consumed long before the projectile reached the muzzle. Indeed nineteen-twentieths of the charge was burned when the projectile had travelled 16.6 ft.

We may also determine k in terms of the travel of projectile by means of the equation

$$k = 1 - \left(1 - \frac{X_o}{\overline{X}_o}\right)^3 \quad \text{(Eq. (46), Chapter IV).}$$

Or, if we wish to know the distance travelled by the projectile when a given fraction of the charge is burned, we have

$$X_o = \overline{X}_o\left\{1 - (1 - k)^{\frac{1}{3}}\right\}$$

As an example, suppose $k = \frac{1}{2}$. Then

$$X_o = \overline{X}_o\left(1 - \left(\frac{1}{2}\right)^{\frac{1}{3}}\right) = \frac{1 - \left(\frac{1}{2}\right)^{\frac{1}{3}}}{N}$$

Applying the value of N found for this round and completing the calculations it will be found that one-half the charge was burned when the projectile had travelled one foot.

The expressions for V and P for this round are

$$V = [3.55681]\sqrt{X_2}$$

and

$$P = \frac{[4.89487]}{(1+x)^{\frac{4}{3}}}$$

The last two formulas give the velocity and pressure upon the supposition that all the powder was converted into gas at temperature of combustion before the projectile had started.

The initial value of P is found by making x zero. Whence

$$P' = 78{,}500 \text{ lbs. per in.}^2$$

Finally we find

$$j = 2338 \text{ lbs.}$$

and

$$v_c = 0.266 \text{ in. per sec.}$$

The following table was computed by the formulas deduced for this round for comparison with the deductions from Sir Andrew Noble's velocity and pressure curves. Unfortunately these curves, as published, are drawn to so small a scale and are so mixed up with other curves that it is difficult to get the velocities and pressures from them with much precision.

NOTE:—The velocities and pressures in the second and third columns were computed by formulas slightly different from those deduced above. But the differences are so small as to be of no account in the discussion.

x	u ft.	Computed Velocities, f. s.	Computed Pressures, lbs. per inch²	Pounds of Powder Burned	V, f. s.	P, lbs. per in.²
0.000	0.000	0.0	0	0.0	0.0	78500
0.001	0.003	12.449	4196	0.716	65.786	78397
0.01	0.030	68.889	12672	2.206	207.40	77466
0.05	0.152	221.52	25254	4.683	457.78	73557
0.1	0.303	359.32	32007	6.357	637.37	69132
0.2	0.607	569.36	37680	8.464	875.21	61560
0.4	1.213	869.33	39439	10.967	1174.0	50122
0.6	1.820	1087.2	37567	12.549	1372.5	41948
0.8	2.427	1257.7	34846	13.686	1520.3	35852
1.0	3.033	1396.6	32050	14.557	1637.0	31153
2.0	6.066	1842.4	21268	17.043	1995.9	18143
3.0	9.100	2093.0	15013	18.225	2192.5	12363
4.0	12.133	2257.1	11164	18.892	2322.4	9181
5.0	15.166	2374.2	8645	19.299	2416.9	7200
5.473	16.6	2419.0	7742	19.435	2453.5	6507
7.121	21.6	2538.0	5500	17.742	2554.9	4809
11.242	34.1	2710.0	2893	19.979	2711.8	2782
15.363	46.6	2806.0	1890	19.999	2806.0	1890

The computed velocities in the third column of this table, corresponding to the travels of projectile in the second column,

agree very well with those deduced from Sir Andrew Noble's velocity curve, from the origin of motion to the muzzle, a distance of 46.6 ft. As the velocities are thus shown to be correct, the pressures in the fourth column are, from their manner of derivation as given in Chapter IV, necessarily correct also. That is, they correspond to the energy of translation of a hundred-pound projectile. In this respect they are more accurate than the pressures given by Sir Andrew's pressure curve which was derived from his velocity curve by graphic methods not sufficiently precise for the great accelerations encountered in ballistic problems.

The writer is indebted to Colonel Lissak, formerly Instructor of Ordnance and Gunnery at West Point, for the accompanying diagram (Fig. 3) of the velocity and pressure curves whose coördinates are given in this table. Many interesting facts may be gleaned from an examination of these curves, and the formulas by which their coördinates were computed.

The two velocity curves v and V are both zero at the origin but immediately separate, attaining their greatest distance apart when the projectile has moved but a short distance. They then approach each other very gradually and become tangent at the point where the powder is all burned,—practically at the muzzle. Both curves are tangent to the axis of ordinates at the origin and parallel to the axis of abscissas at infinity. The pressure curve p begins at the origin, attains its maximum when the projectile has traveled about 15 inches, changes direction of curvature when u is about six feet and meets the axis of abscissas at infinity. The pressure curve P is convex toward the axis of abscissas throughout its whole extent. It lies above the curve p from $u = 0$ to $u = 30$ inches (about), then passes below p and the two curves become tangent at the point where the powder is all consumed. Finally the areas under the curves p and P are equal.

Example. 1.—Suppose the charge in the example under

consideration to be increased from 20 to 25 lbs. Deduce the equations for velocity and pressure.

In solving this example, we will compute the new constants M, M', N and N' by equations (80) to (83), Chapter IV; and as the charge is increased by 25 per cent., a new value of f must be found by (90) and (90′). For a six-inch gun we will take $n = \frac{1}{3}$, provisionally; and since the weight of the projectile remains the same, n' must be zero. We therefore have

$$K = 86.12,$$

and the new value of f is 2518 lbs.

The new values of a and z_o for $\tilde{\omega} = 25$ lbs., are

$$\log a = 0.13321$$
$$\log z_o = 0.44277 \qquad \therefore \quad z_o = 2.772 \text{ ft.}$$

Applying these numbers in the equations above mentioned we find for a charge of 25 lbs.,

$$\log M = 6.73881$$
$$\log M' = 5.03633$$
$$\log N = 9.01885$$
$$\log N' = 7.56058$$

which give the equations required.

These constants give $p_m = 55676$ lbs., and a velocity of 2841 f. s., for a travel of 16.6 ft. That is, an increase of 5 lbs. in the charge increases the maximum pressure 16,800 lbs. per in.², and the velocity at 16.6 ft. travel, 425 f. s. Taking the reciprocal of N gives

$$\log \overline{X}_o = 0.98115$$

and from the table,

$$\overline{x} = 18.1425$$

and

$$\overline{u} = 18.1425 \times 2.772 = 50.29 \text{ ft.}$$

10

The limiting velocity and fraction of charge burned are given by the equations

$$V_1^2 = \frac{M\overline{X}_o}{3} \text{ and } k = \frac{v^2}{V_1^2 X_2}$$

Therefore

$$\log V_1^2 = 7.24284$$

and

$$y = k\,\tilde{\omega} = [4.15510 - 10]\frac{v_2}{X_2}$$

From this last formula we find when $u = 16.6$ ft., $y = 24.18$ lbs.

The pressure at this point is found to be 10480 lbs. per in.^{2} It is interesting to compare these results with those found with a charge of 20 lbs.

In order to lessen the maximum pressure the grains must be increased in size and thus diminish the initial burning surface. Suppose we increase the size of the cubes from $0''.3$ to $0''.5$ on a side. Determine the equations of velocity and pressure for a charge of 25 lbs. An examination of equations (80) to (82), Chapter IV, will show that when the only change in the data is in the thickness of web the new values of M, M', and N will be found by multiplying the previously determined values of these constants by the ratio of the web thicknesses,—in this case by 0.6. We therefore have for 25 lbs. of $0''.5$ cubes

$$\begin{aligned} \log M &= 6.51696 \\ \log M' &= 4.81448 \\ \log N &= 8.79700 - 10 \\ \log N' &= 7.11688 - 10 \end{aligned}$$

From these we get

$$p_m = 38437 \text{ lbs. per in.}^2$$

and

$$v = 2571 \text{ f. s., for } u = 16.6 \text{ ft.}$$

The measured velocity for this travel of projectile was 2416

f. s., with a charge of 20 lbs. of $0''.3$ cubes. Therefore by increasing the weight of charge 5 lbs., and at the same time enlarging the grain from $0''.3$ to $0''.5$ on a side the velocity is increased 155 f. s.,—and this without increasing the maximum pressure, though the mean pressure is, of course, considerably increased.

The pressure for $u = 16.6$ ft., with a charge of 20 lbs. of the smaller grains, was 7741 lbs.; and with a charge of 25 lbs. of the larger grains, the pressure for the same travel would be 11181 lbs. The powder actually burned during this travel of projectile is a little more in this latter case than in the former, and the space in which it has been confined during its expansion is less, both of which facts account for the greater work performed.

From equation (19′), Chapter III, it follows that for two equal charges made up of grains of the same form and differing only in their size, the entire initial surfaces of the two charges vary inversely as the thickness of web. Therefore the initial surface of the charge of $0.''5$ grains is $\frac{3}{5}$ of the initial surface of the same charge of $0.''3$ grains. This accounts for the two charges giving the same maximum pressure. It may be remarked that the same results would have been obtained if the grains had been spherical instead of cubical.

Application to Multiperforated Grains.—A peculiar difficulty arises in the application of any system of interior ballistic formulas to multiperforated grains from the fact that they do not retain their original form until completely consumed—as do all other forms of grain in use,—but each grain breaks up, when the web thickness proper is burned through, into twelve slender rods, or "slivers," which burn according to a different law; and thus two independent sets of formulas become necessary to represent what actually takes place in the gun. It was previously sought to overcome this difficulty by supposing the web thickness to be slightly increased so as to

satisfy the equation of condition

$$\alpha\,(1 + \lambda - \mu) = 1$$

and thus ignoring the slivers.* This method represents quite satisfactorily the actual circumstances of motion so long as the grains retain their original form, but not afterward. It assumes that the slivers are all burned with the fictitious web thickness; that is, when, in all our guns, the projectile has performed approximately half its travel in the bore; while it is certain that in most cases with our service powders they are not completely consumed when the projectile leaves the bore. It is necessary, therefore, to divide the entire combustion of the grain into two periods and to deduce formulas that shall represent the law of burning, as well as the circumstances of motion, for each period.

From equation (22), Chapter III, we have, for m.p. grains

$$k = \frac{y}{\bar{\omega}} = \alpha\frac{l}{l_o}\left\{1 + \lambda\frac{l}{l_o} - \mu\frac{l^2}{l^2_o}\right\}$$

which gives the fraction of the charge consumed when any thickness l of the web has been burned, and this without any reference to the law of burning. When $l = l_o$, that is, when the entire web thickness has been burned, this equation becomes

$$k' = \alpha\,(1 + \lambda - \mu)$$

in which k' is the fraction of the charge less the slivers. If we substitute for α, λ and μ their values for any of our m.p. grains, we shall find for this critical point,

$$k' = 0.85 \text{ (about)},$$

and therefore the slivers constitute approximately 15 per cent. of the charge. These slivers burn according to another law. We may regard them as slender cylinders whose form characteristics are very approximately

$$\alpha = 2,\ \lambda = -1,\ \mu = 0.$$

* See Journal U. S. Artillery, vol. 24, p. 196, and vol. 26, pp. 141 and 276.

We will now deduce formulas for each period of burning.

Designate all symbols referring to the point where the grains are converted into slivers by an accent, and those relating to the muzzle—including M, M' and N—by a subscript m.

Equation (11), Chapter IV, becomes, by suitable reductions,

$$K = [8.56006 - 10]\frac{v_c \sqrt{a\,w\,\bar{\omega}}}{d^2\, l_o} \quad . \quad . \quad . \quad . \quad (44)$$

in which v_c is the velocity of combustion under atmospheric pressure and l_o one-half the web thickness. From (12), Chapter IV, we have

$$l/l_o = K\, X_o$$

which, when the web thickness is burned through, gives for this critical point,

$$K\, X'_o = 1$$

Therefore from (44)

$$X'_o = [1.43994]\frac{d^2\, l_o}{v_c \sqrt{a\,w\,\bar{\omega}}} \quad . \quad . \quad . \quad . \quad . \quad (45)$$

which gives X'_o when v_c and l_o are known. Also

$$v_c = [1.43994]\frac{d^2\, l_o}{X'_o \sqrt{a\,w\,\bar{\omega}}} \quad . \quad . \quad . \quad . \quad (46)$$

and

$$l_o = [8.56006 - 10]\frac{v_c\, X'_o \sqrt{a\,w\,\bar{\omega}}}{d^2} \quad . \quad . \quad . \quad (46')$$

While the slivers are burning we have from (49), Chapter IV,

$$X_o = \frac{K}{2\, N_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

in which X_o refers to any travel between u' and u_m and

$$K = 1 - (1 - k)^{\frac{1}{2}}$$

If we know the values of X_o and k for any point, we can

determine N_m by the equation

$$N_m = \frac{K}{2\,X_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (48)$$

Therefore at the point of breaking up into slivers (48) becomes

$$N_m = \frac{K'}{2\,X'_o} \text{ or } X'_o = \frac{K'}{2\,N_m} \quad . \quad . \quad . \quad . \quad . \qquad (49)$$

The fraction k' which enters into K' can be computed for a grain of given dimensions by (21), Chapter III; and log K' can be taken from Table II with k' as the argument. Therefore when N_m is known X'_o can be found from (49), and then x', taken from Table I with X'_o as the argument, locates the point where the grains become slivers by the equation

$$u' = x'\,z_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (50)$$

In order to determine N_m it is necessary to assume a value for k_m, or the fraction of the entire charge burned at the muzzle, and check this assumed value by the given maximum (crusher-gauge) pressure. By (45), Chapter IV, we have,

$$V_1^2 = \frac{v_m^2}{k_m\,X_{2m}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (51)$$

by means of which V_1 can be determined from muzzle data. The constants M, N and N' to be used in the velocity and pressure formulas from $u = 0$ to u' are given by the formulas

$$M = \frac{\alpha\,V_1^2}{X'_o},\quad N = \frac{\lambda}{X'_o} \text{ and } N' = \frac{\mu\,N^2}{\lambda^2} \quad . \quad . \qquad (52)$$

Finally the value of M_m for the travel from u' to u_m is given by the formulas, deduced from (3),

$$M_m = \frac{\alpha}{\lambda}\,N_m\,V_1^2 = 4\,N_m\,V_1^2 \quad . \quad . \quad . \quad . \qquad (53)$$

As an example of this method we will take the mean crusher-gauge pressure and muzzle velocity of five shots fired March 14,

1905, with the 6-inch Brown wire gun, by the Board of Ordnance at Sandy Hook. The gun had been previously fired twenty-six times with charges varying in weight from 32 lbs. to 69 lbs., and at this time was very little eroded. The gun data are as follows:

$$V_c = 3120 \text{ c. i.}$$
$$d = 6 \text{ inches}$$
$$u_m = 252.5 \text{ inches (total travel in bore).}$$

The firing charge for these five shots was 70 lbs. of nitro-cellulose powder, with 8 ounces of black rifle powder at each end of the cartridge for a primer. As it is impossible to isolate the action of each kind of powder, we will consider the charge in its entirety and take $\tilde{\omega} = 71$ lbs.* The projectiles varied slightly in weight from 100 lbs. (about one-quarter of one per cent.); but no material error will result if we make $w = 100$ lbs. The mean muzzle velocity (v_m) was 3330.4 f. s., and the mean crusher-gauge pressure (p_m) was 42497 lbs. per in.^{2} The charges were made up of m.p. grains designed for an 8-inch rifle, and of the following dimensions: $R = 0''.256$; $r = 0''.0255$; $m = 1''.029$. And, therefore, $l_o = 0''.044875$; $\alpha = 0.72667$; $\lambda = 0.19590$; $\mu = 0.02378$; $k' = 0.85174$.

The granulation of this powder is 89 grains to the pound. The volume of a single grain computed by (15), Chapter III, is 0.197144 c. i.; whence by (23′), Chapter III, $\delta = 1.5776$. From these data are found by methods already fully illustrated,

$$\Delta = 0.6299$$
$$\log a = 9.97940 - 10$$
$$\log z_o = 1.82144 \quad \therefore\ z_o = 66.289 \text{ in.}$$
$$x_m = 3.8091$$
$$\log X_{om} = 0.74029$$

* Gossot recommends to increase the weight of charge by one-third that of the igniter. But there is no practical difference in the results by the two methods.

We will assume $k_m = 0.973$. Therefore from Table II, $\log K_m = 9.92204 - 10$, and $\log K' = 9.78885 - 10$. From (48) we have

$$N_m = \frac{K_m}{2\,X_{om}} = \frac{K'}{2\,X'_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

whence

$$X'_o = \frac{K'\,X_{om}}{K_m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

$$\therefore \log X'_o = 0.60710$$

We now find from the preceding formulas,

$$\begin{aligned} \log V_1{}^2 &= 7.44669 \\ \log M &= 6.70093 \\ \log M' &= 4.69894 \\ \log N &= 8.68493 - 10 \\ \log N' &= 7.16201 - 10 \end{aligned}$$

Substituting these values of M', N and N' in equation (51), Chapter IV, gives $p_m = 42521$ lbs. per in.², differing insensibly from the mean crusher-gauge pressure. The assumed value of k_m is therefore correct. We now find from (54) and (53),

$$\begin{aligned} \log N_m &= 8.88072 - 10 \\ \log M_m &= 6.92947 \end{aligned}$$

The value of x' taken from Table 1, by means of $\log X'_o$, is

$$x' = 1.757.$$

Therefore $u' = 1.757 \times 66.289 = 116.47$ inches.

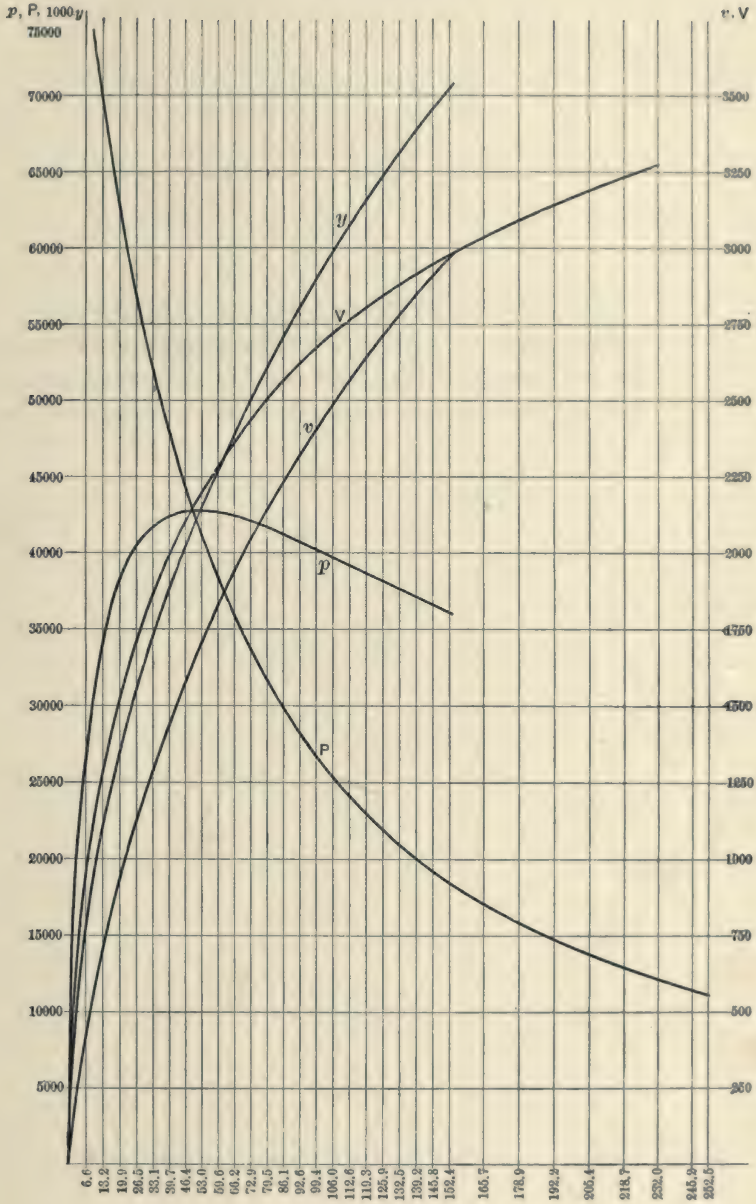
The two sets of equations for velocity and pressure for a charge of 70 lbs., and primer of 1 pound, are:—

From $u = 0$ to $u' = 116.47$ inches:

$$v^2 = [6.70093]\,X_1 \left\{1 + [8.68493 - 10]\,X_o - [7.16201 - 10]\,X^2_o\right\}$$
$$p = [4.69894]\,X_3 \left\{1 + [8.68493 - 10]\,X_4 - [7.16201 - 10]\,X_5\right\}$$

From $u' = 116.47$ in. to muzzle:—

$$v^2 = [6.92947]\,X_1 \left\{1 - [8.88072 - 10]\,X_o\right\}$$
$$p = [4.92748]\,X_3 \left\{1 - [8.88072 - 10]\,X_4\right\}$$


p, P, 1000y
v, V
75000
70000
65000
60000
55000
50000
45000
40000
35000
30000
25000
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15000
10000
5000
3500
3250
3000
2750
2500
2250
2000
1750
1500
1250
1000
750
500
250
y
V
v
p
P
6.6
13.2
19.9
26.5
33.1
39.7
46.4
53.0
59.6
66.2
72.9
79.5
86.1
92.6
99.4
106.0
112.6
119.3
125.9
132.5
139.2
145.8
152.4
165.7
178.9
192.2
205.4
218.7
232.0
245.2
252.5

FIG. 4.

The X functions for the travel u' are

$$\log X'_0 = 0.60710$$
$$\log X'_1 = 0.06473$$
$$\log X'_2 = 9.45763 - 10$$
$$\log X'_3 = 9.79332 - 10$$
$$\log X'_4 = 0.76498$$
$$\log X'_5 = 1.48763$$

Both formulas for velocity give the same velocity for the travel u', namely $v' = 2614$ f. s. The pressure at this point by the first formula is 38431 lbs.; and by the second 29324 lbs. per in.^{2} The discontinuity shown by the two curves P and p (see Fig. 4), at the travel u', where the grains break up into slivers is due to the sudden diminution of the surface of combustion of the grains at this point, whereby the rate of evolution of gas and heat suddenly falls and with it also the pressure. In this particular example the initial burning surface of each grain is 3.2 in.^{2}, and goes on increasing until at the point of breaking up the vanishing surface is 4.2 in.^{2} It then suddenly falls to about 1.5 in.^{2}, which is approximately the surface of the twelve slivers. Of course there is no such absolutely abrupt fall in the pressure as is indicated by the two pressure formulas. Neither can it be supposed that all the grains maintain their original form until the web thickness is completely burned. Nevertheless the two pressure formulas give very approximately the average pressure at or near this point. It might be possible to connect the two pressure curves by another curve of very steep descent; but this is hardly necessary.

The characteristics f and v_c are $f = 1418$ and $v_c = 0.134$. These characteristics, computed with the firing data of an 8-inch gun, were found on page 105 to be 1397 and 0.136, respectively.

The expression for y (powder burned) is by (45), Chapter IV,

$$y = [4.40457] \frac{v^2}{X_2}$$

For the travel u', this formula gives $y' = 60.472$ lbs. $= 71\,k'$.

At the muzzle, by the above formula, $y_m = 69.085$ lbs. $= 71\,k_m$.

If it should be found in any case that the powder was all burned in the gun, it would be necessary to compute X'_o by the formula

$$X'_o = K'\,\overline{X}_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

In this case we should assume a value for $\overline{X}_o$ (or $\overline{x}$), and compute the maximum pressure for comparison with the crusher-gauge pressure, following the same steps as before.

We will consider a few additional problems illustrative of this method of treating m.p. grains.

Problem 1.—What must be the dimensions of the grains in the example just considered in order that the combustion of the entire charge may be completed at the muzzle? Also what would be the muzzle velocity and maximum pressure?

In solving this problem we must first consider the second period of combustion, namely, that of the slivers. It has already been shown that for a charge of 71 lbs., $\log V_1^2 = 7.44669$. We also have in this case, since $k_m = 1$,

$$N_m = \frac{1}{2\,\overline{X}_o} \text{ (by (48))}$$

$\overline{X}_o$ being the muzzle value of X_o; and by (53),

$$M_m = 4\,N_m\,V_1^2$$

We thus find for the second period of combustion,

$$\left.\begin{array}{l} v^2 = [7.00743]\,X_1 \left\{1 - [8.95868 - 10]\,X_o\right\} \\ p = [5.00544]\,X_3 \left\{1 - [8.95868 - 10]\,X_4\right\} \end{array}\right\} A$$

The muzzle velocity by the above equation is 3376 f. s., an increase of 46 f. s., due to the combustion of the entire charge in the gun. The muzzle pressure is 11433 lbs. per in.^{2}

In order to deduce equations for velocity and pressure for the first period of combustion, it will be necessary to determine

the value of k' from which to compute X'_o and $2\,l_o$. Suppose we adopt grains for which $R/r = 11$ and $m/l_o = 30$.

By the method given in Chapter III, we find for grains having these ratios, $\alpha = \frac{199}{285}$, $\lambda = \frac{48}{199}$, $\mu = -\frac{4}{199}$ and $k' = \frac{81}{95}$. For this value of k' we find from Table II, $\log K' = 9.78966$; and since $K_m = 1$, we have from (55),

$$X'_o = K'\,X_{om}$$

which gives

$$\log X'_o = 0.52995.$$

By interpolation from Table 1, we find $x' = 1.15217$, and then $\log X'_1 = 9.88302 - 10$, $\log X'_3 = 9.83966 - 10$, $\log X'_4 = 0.67980$ and $\log X'_5 = 1.32095$. Next by equations (52), and equation (61), Chapter IV, we deduce the following equations for velocity and pressure, which apply from $u = 0$ to $u' = 1.1522 \times 66.289 = 76.38$ inches:

$$\left.\begin{aligned} v^2 &= [6.76075]\,X_1\{1+[8.85245-10]\,X_o-[7.24333-10]\,X^2_o\} \\ p &= [4.75876]\,X_3\{1+[8.85245-10]\,X_4-[7.24333-10]\,X_5\} \end{aligned}\right\} B$$

Both sets of equations, A and B, give the same value to v', namely, 2319 f. s., while the pressures at u' by the two equations are, respectively, 51723 and 39560 lbs. per in.^{2},—a drop of more than 12000 lbs. when the grains break up into slivers. The maximum pressure (taking $x = 0.8$) is 52428 lbs. per in.^{2}

The dimensions of the grains have yet to be determined. We have found for this powder $v_c = 0.134$ in. per sec. Substituting this and the value of $\log X'_o$, given above, in (46′) gives,

$$l_o = 0.038 \text{ in.}$$

and then

$$r = l_o/2 = 0.019 \text{ in.}$$
$$R = 11\,r = 0.209 \text{ in.}$$
$$m = 30\,l_o = 1.14 \text{ in.}$$

A grain of these dimensions fulfils all the conditions of the problem. These calculations show in a striking manner the great effect which minute variations (scarcely measurable) in the dimensions of m.p. grains have upon the maximum pressure, increasing it in this case by 10,000 lbs. per in.[2]

The cause of this great increase in the maximum pressure is that the initial surface of combustion of the charge of the smaller grains is about 15 per cent. greater than that of the original grains, as is easily shown by equation (26′), Chapter III.

Problem 2.—What must be the dimensions of the grains of a charge of 71 lbs., in order that the burning of the web may be completed at the muzzle? Also determine the circumstances of motion.

To solve this problem we obviously have $u' = u_m$; and therefore $x' = x_m = 3.8091$. As all the X functions relate to the muzzle only, we may drop the accents. We have from Table 1, $\log X_o = 0.74029$, $\log X_1 = 0.35048$, $\log X_2 = 9.61018$, $\log X_3 = 9.65311$, $\log X_4 = 0.91582$, and $\log X_5 = 1.78077$. Substituting the value of $\log X_o$ in (46), and making use of the known value of v_c, we find that for the new grains,

$$l_o = 0''.06098$$

Therefore, as in Problem 1, $r = \dfrac{l_o}{2} = 0''.03049$

$$R = 5.5\, l_o = 0''.33539$$
$$m = 30\, l_o = 1''.8294$$
$$k' = 0.85263 = k_m$$

Since the limiting velocity V_1 is independent of the dimensions of the grains, we have as before, $\log V_1^2 = 7.44669$; and this, with the known values of α, λ and μ, substituted in equations (52), gives M, N and N'. We thus derive the following equations for velocity and pressure for a charge of 71 lbs. of these particular grains.

$$v^2 = [6.55041]\, X_1 \left\{ 1 + [8.64211 - 10]\, X_o - [6.82262 - 10]\, X_o^{\,2} \right\}$$
$$p = [4.54842]\, X_3 \left\{ 1 + [8.64211 - 10]\, X_4 - [6.82262 - 10]\, X_5 \right\}$$

From these formulas we get the following information:

Muzzle velocity, 3118 f. s.

Maximum pressure, 29897 lbs. per in.^{2}

Muzzle pressure, 21014 lbs. per in.^{2}

Powder burned in gun, 60.5 lbs. = 71 k'.

The maximum pressure is quite moderate, owing to the thickness of web which gives an initial surface of combustion but 71 per cent. of that of the original grains. The pressure is well sustained to the muzzle, where it would be considered excessive for all except wire-wound guns.

If we suppose the length of the grains to be twelve times the web thickness we should have $\alpha = \frac{163}{228}$, $\lambda = \frac{36}{163}$, $\mu = -\frac{4}{163}$, $k' = \frac{65}{76}$, and $m = 0.916$ in. Then, as before,

$$\log M = 6.56065$$
$$\log M' = 4.55866$$
$$\log N = 8.60383 - 10$$
$$\log N' = 6.90929 - 10$$

These constants give—

Muzzle velocity, 3124 f. s.

Maximum pressure, 30163 lbs. per in.^{2}

Muzzle pressure, 20875 lbs. per in.^{2}

Powder burned in gun, 60.72 lbs. = 71 k'

The initial surface of combustion of the shorter grain is about 2.4 per cent. greater than that of the longer grain, which fact is shown in the maximum pressures.

Problem 3.—Suppose the powder we have been considering to be moulded into cylinders with an axial perforation.

If the length of the grain is 50 inches (approximately the length of the cartridge), and the diameter of the axial perforation

one-twentieth of an inch, what must be the diameter of the grain and thickness of web in order that a charge of 71 lbs. may all be burned just as the shot leaves the muzzle? Also determine the equations for velocity and pressure.

We have already found the thickness of web satisfying the conditions of the problem to be 0''.12196. (See Problem 1.) Therefore, by means of the formulas pertaining to this form of grain given in Chapter III, we find the diameter of the grains to be 0''.294 and

$$\alpha = 1.0024392$$
$$\lambda = 0.0024333$$
$$\mu = 0$$

Since $\log \overline{X}_o = 0.74029$ and $\log V_1^2 = 7.44669$, we find

$$v^2 = [6.70746]\, X_1 \left\{ 1 - [6.64590 - 10]\, X_o \right\}$$
$$p = [4.70547]\, X_3 \left\{ 1 - [6.64590 - 10]\, X_4 \right\}$$

which are the equations required. The muzzle velocity and maximum pressure by these formulas are

$$v_m = 3376 \text{ f. s.}$$
$$p_m = 37040 \text{ lbs. per in.}^2$$

This latter, on account of the smallness of N, occurs when $x = 0.64$. The muzzle pressure is 22750 lbs. per in.^{2}

A comparison of these results with those deduced in Problem 1 shows the great superiority of the uniperforated grain over the multiperforated grain so far as maximum pressure is concerned. The muzzle velocity is the same in both cases since the same weight of powder was burned in the gun. But the maximum pressure given by the m.p. grains is more than 15,000 lbs. greater, and the muzzle pressure 11,000 lbs. less than with the u.p. grains. For these latter grains the pressure is remarkably well sustained from start to finish.

The monomial formulas for velocity and pressure for this example are easily found to be

$$v = [3.35320] \sqrt{X_1}$$

and

$$p = [4.70441] X_3$$

The first of these gives the same value for the muzzle velocity as the complete formula; while the second gives maximum and muzzle pressures differing about 0.1 per cent. of their former values.

During the test-firing of the 6-inch Brown wire-wound gun at Sandy Hook, shots were fired with charges varying from 32¼ lbs. to 75 lbs., thus enabling us to determine whether our formulas have any predictive value. Unfortunately the object of the firing was simply to test the endurance of the gun and no special effort was made to give to the results any scientific value. Many of the recorded velocities and pressures are inconsistent with each other as when, more than once, an increase of charge gave a diminished velocity and pressure. Some of the recorded muzzle velocities are so manifestly wrong that they cannot be used in getting averages. They suggest that the chronograph velocities were not always reduced to the muzzle.

We will compute the new values of f due to a change in the weight of charge by (88), Chapter IV, taking $\bar{\omega}_o = 71$ lbs., and $f_o = 1418$ lbs. per in.², and for a six-inch gun, $n = \frac{1}{3}$. We therefore have

$$f = [2.53451]\, \bar{\omega}^{\frac{1}{3}}$$

To determine X'_o, we have

$$X'_o = [1.43994] \frac{d^2 l_o}{v_c \sqrt{a\, w\, \bar{\omega}}}$$

which, by substituting the known values of d, l_o, v_c and w, reduces to

$$X'_o = \frac{[1.52243]}{\sqrt{a\, \bar{\omega}}}$$

N_m is given by (54), which easily reduces to (since $\log K' = 9.78885 - 10$)

$$N_m = [7.96539 - 10] \sqrt{a\, \bar{\omega}} \quad . \quad . \quad . \quad . \quad (a)$$

Next we have from equation (58), Chapter IV, substituting for f its value given above and for w its value, 100 lbs.,

$$V_1^2 = [4.97834]\, \bar{\omega}^{\frac{7}{8}}$$

and lastly from (53),

$$M_m = 4\, N_m V_1^2 = [3.54579]\, a^{\frac{1}{2}}\, \bar{\omega}^{\frac{11}{8}} \quad . \quad . \quad . \quad (b)$$

The following table computed by these formulas shows the agreement between the observed and computed velocities for a range of charges between 75 lbs. and 33¼ lbs. The differences in the last column follow no apparent law and are unimportant.

$\bar{\omega}$ lbs.	x_m	$\log M_m$	$\log N_m$	Observed Velocity	Computed Velocity	O.−C.
75.0	3.9574	6.95290	8.87242-10	3455	3477	−22
74.5	3.9383	6.95008	8.87347	3422	3459	−37
73.5	3.9005	6.94436	8.87557	3402	3423	−21
72.5	3.8635	6.93849	8.87764	3380	3385	− 5
71.0	3.8091	6.92947	8.88072	3330	3330	0
69.0	3.7392	6.91693	8.88474	3254	3257	− 3
68.0	3.7052	6.91047	8.88672	3236	3220	16
59.0	3.4244	6.84548	8.90384	2879	2888	− 9
49.625	3.1742	6.76170	8.92037	2484	2536	−52
33.25	2.8146	6.55588	8.94643	1913	1896	17

The two sets of equations for velocity and pressure for the charge of 75 lbs. are:

From $u = 0$ to $u' = 117.43$ inches:—

$$v^2 = [6.72436]\, X_1 \left\{1 + [8.67663 - 10]\, X_o - [7.14541 - 10]\, X_o^2\right\}$$
$$p = [4.73896]\, X_3 \left\{1 + [8.67663 - 10]\, X_4 - [7.14541 - 10]\, X_5\right\}$$

From $u' = 117.43$ in. to muzzle:

$$v^2 = [6.95290]\, X_1] \left\{1 - [8.87242 - 10]\, X_o\right\}$$
$$p = [4.96750]\, X_3 \left\{1 - [8.87242 - 10]\, X_4\right\}$$

By the first equation for pressure we find $p_m = 46509$ lbs. per in.[2] And by the second, muzzle pressure = 15375 lbs. per in.[2] Both expressions for velocity give $v' = 2744$ f. s.

For a charge of 62 lbs., the two sets of equations are

From $u = 0$ to $u' = 114.54$ inches:—

$$v^2 = [6.63999]\, X_1 \left\{1 + [8.70249 - 10]\, X_o - [7.19713 - 10]\, X_o^2\right\}$$
$$p = [4.60288]\, X_3 \left\{1 + [8.70249 - 10]\, X_4 - [7.19713 - 10]\, X_5\right\}$$

From u' to muzzle:

$$v^2 = [6.86853]\, X_1 \left\{1 - [8.89828 - 10]\, X_o\right\}$$
$$p = [4.83142]\, X_3 \left\{1 - [8.89828 - 10]\, X_4\right\}$$

These formulas give a muzzle velocity of 3,000 f. s., with a maximum pressure of 34,263 lbs., and a muzzle pressure of 11,784 lbs. per in.[2] It would seem as if these last results are all that could be desired for a 6-inch gun.

Application to the Fourteen-Inch Rifle

The 14-inch rifle was designed by the Ordnance Department to give a "muzzle velocity of 2,150 f. s. to a projectile weighing 1,660 lbs., with a charge of nitrocellulose powder of about 312 lbs., and with a maximum pressure not to exceed 38,000 lbs. per square inch." The gun has a powder-chamber capacity of 13,526 cubic inches and a travel of projectile in the bore of 413.85 inches. The type gun has been fired to date 55 times with charges varying from 102½ to 328 lbs., producing muzzle velocities ranging from 901 to 2,252 f. s., and crusher-gauge pressures from 4,875 to 46,078 lbs. per in.[2], this latter with a charge of 326 lbs.

The powder employed was "International Smokeless powder, lot 1, 1906, for 12-inch gun." The grains were cylindrical multiperforated (7 perforations), of the following dimensions:

Outside diameter, 0.826 in.
Diameter of perforations, 0.0815 in.
Length, 1.883 in.
Thickness of web, 0.145375 in.
These dimensions give:

$$\begin{aligned} \alpha &= 0.71584 \\ \lambda &= 0.20974 \\ \mu &= 0.02151 \\ k' &= 0.85058 \\ \log K' &= 9.78778\text{–}10. \end{aligned}$$

The granulation of the powder is 20.6 grains to the pound, which by (24′), Chapter III, makes the density (δ) 1.4291.

We will base our calculations on round No. 55, fired January 23, 1911, with a charge of 328 lbs. of nitrocellulose powder plus an "igniter" of 9 lbs. of rifle, or saluting, powder. This round affords the following data:

$$\begin{aligned} \tilde{\omega} &= 337 \text{ lbs.} \\ w &= 1664 \text{ lbs.} \\ v_m &= 2252 \text{ f.s.} \\ p_m &= 43640 \text{ lbs. per in.}^2 \end{aligned}$$

The preliminary calculations give

$$\begin{aligned} \Delta &= 0.68965 \\ \log a &= 9.87523\text{–}10 \\ \log z_o &= 1.65768 \\ x_m &= 9.1025 \\ \log X_{om} &= 0.87855 \\ \log X_{1m} &= 0.60885 \end{aligned}$$

By a few trials it will be found that the observed values of v_m and p_m are satisfied when $k_m = 0.953$ and therefore from Table II, $\log K_m = 9.89388\text{–}10$. We also find $\log X'_o = 0.77245$, $\log V_1^2 = 6.99575$, $x' = 4.6354$ and $u' = 210.75$ inches.

The equations of the velocity and pressure curves are found to be

From $u = 0$ to $u' = 210.75$ in.:

$$v^2 = [6.07812]\, X_1 \left\{1 + [8.54923\text{–}10]\, X_o - [6.78775\text{–}10]\, X_o^2\right\}$$
$$p = [4.72511]\, X_3 \left\{1 + [8.54923\text{–}10]\, X_4 - [6.78775\text{–}10]\, X_5\right\}$$

From $u' = 210.75$ in. to muzzle:

$$v^2 = [6.31211]\, X_1 \left\{1 - [8.71430\text{–}10]\, X_o\right\}$$
$$p = [4.95910]\, X_3 \left\{1 - [8.71430\text{–}10]\, X_4\right\}$$

Both of the velocity formulas give $v' = 1921$ f. s. The first formula for pressure gives $p' = 27457$ and the second 19,772 lbs. per in.^{2} The muzzle pressure comes out 9,485 lbs. per in.^{2}

This round makes the powder characteristics, by (64) and (67), Chapter IV,

$$f = 1759.7$$
$$v_c = 0.10214$$

For computing the velocity and pressure constants when the charge varies, we will consider v_c constant and assume f to vary directly as the weight of charge. That is, we will compute f by the formula

$$f = \frac{1759.7\tilde{\omega}}{337} = [0.71781]\, \tilde{\omega} \quad . \quad . \quad . \quad . \quad (a)$$

Equation (69), Chapter IV, becomes, by substituting the values of d^2, l_o and v_c,

$$X'_o = \frac{[3.58446]}{\sqrt{aw\tilde{\omega}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Also (58), Chapter IV, becomes, by employing the expression for f given above,

$$V_1^2 = \frac{[5.16164]\tilde{\omega}^2}{w'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

We then have

$$M = \frac{\alpha V_1^2}{X'_o} = [1.43200]\, \bar{\omega} \left(\frac{a\bar{\omega}}{w}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (d)$$

$$N = \frac{\lambda}{X'_o} = [5.73722-10]\, \sqrt{aw\bar{\omega}} \quad . \quad . \quad . \quad . \quad (e)$$

and

$$N' = \frac{\mu}{\lambda^2} N^2 = [9.68929-10]\, N^2 \quad . \quad . \quad . \quad . \quad (f)$$

By (49), we have

$$N_m = \frac{K'}{2X'_o}$$

Combining this with the expression for N we have

$$N_m = \frac{K'N}{2\lambda} = (0.16507)\, N \quad . \quad . \quad . \quad . \quad . \quad (g)$$

Finally we have from (53)

$$M_m = 4\, N_m\, V_1^2 = \frac{2K'M}{\alpha} = (0.23399)\, M \quad . \quad . \quad (h)$$

The following table gives the computed muzzle velocities and maximum pressures for certain charges, computed by these formulas, together with the observed velocities and crusher-gauge pressures for comparison:

$\bar{\omega}$ lbs.	w lbs.	Observed Velocity, f. s.	Computed Velocity, f. s.	O.–C. f. s.	Observed Pressure, lbs. per in.^{2}	Computed Pressure, lbs. per in.^{2}	O.–C.
337	1664	2252	2252	0	43640	43628	12
335	1660	2238	2240	– 2	42811	42944	–133
334	1660	2232	2232	0	42877	42637	240
284	1662½	1857	1871	–14	25530	29142	–3612
263	1660	1738	1724	14	21190	24431	–3241
239	1660	1567	1556	11	16795	19704	–2909

The greatest difference between the observed and computed muzzle velocities is considerably less than one per cent. and

may be disregarded. The same is true of the differences of the observed and computed maximum pressures of the first three charges. Then, as the charges are greatly reduced, these differences are largely increased. This may be accounted for if the same kind of copper cylinders were employed for all the charges.

For a charge of 314 lbs. of service powder and an igniter of 9 lbs. of rifle powder, making $\tilde{\omega} = 323$ lbs., and density of loading 0.661, the equations for velocity and pressure are as follows:

From $u = 0$ to $u' = 208.75$ inches.

$$v^2 = [6.05003]\, X_1 \left\{1 + [8.55696-10]\, X_o - [6.80321-10]\, X^2{}_o\right\}$$
$$p = [4.67944]\, X_3 \left\{1 + [8.55696-10]\, X_4 - [6.80321-10]\, X_5\right\}$$

From $u' = 208.75$ inches to muzzle.

$$v^2 = [6.28402]\, X_1 \left\{1 - [8.72203-10]\, X_o\right\}$$
$$p = [4.91343]\, X_3 \left\{1 - [8.72203-10]\, X_4\right\}$$

These formulas give

$$f = 1686.6$$
$$\text{Muzzle velocity} = 2152 \text{ f. s.}$$
$$\text{Maximum pressure} = 39351 \text{ lbs. per in.}^2$$

This muzzle velocity is that for which the gun was designed, but the maximum pressure is about 3½ per cent. greater. The muzzle pressure comes out 8714 lbs. per in.^{2}

Example.—Suppose the volume of the powder chamber to be increased (as is proposed by the Ordnance Department) to 15,000 cubic inches, by lengthening the chamber 6.65 inches, thereby reducing the travel of the projectile to 407.2 inches. If the density of loading remain 0.661, what would be the charge, the muzzle velocity, and maximum pressure?

Answers:

$$\tilde{\omega} = 349.2 + 9 = 358.2 \text{ lbs.}$$
$$p_m = 41683 \text{ lbs. per in.}^2$$
$$M.\,V. = 2233.5 \text{ f. s.}$$

With a charge of 337 lbs. of service powder and an igniter of 9 lbs. of black powder, we should get, with the lengthened chamber, a muzzle velocity of 2150 f. s., with a maximum pressure of about 38,400 lbs. per in.[2] These results are practically those sought for in designing the present 14-inch gun.

Example 2.—Suppose, instead of enlarging the powder chamber of the 14-inch gun, we lengthen the grains of powder, and employ the ratios $R/r = 11$ and $m/l_o = 200$. These ratios give, as is shown in Chapter III,

$$\alpha = \frac{1219}{1900} = 0.64158$$

$$\lambda = \frac{388}{1219} = 0.31829$$

$$\mu = -\frac{4}{1219} = -0.00328$$

$$k' = \frac{1603}{1900} = 0.84368$$

$$\log K' = 9.78150 - 10. \quad \text{(By Table II.)}$$

Employing these grains, what muzzle velocity and maximum pressure may be expected with a charge of 314 lbs. of service powder and an igniter of 9 lbs. of black powder, in the gun as it is now, where $V_c = 13526$ c. i., and $u_m = 413.85$ in.?

The preliminary calculations give:

$$\begin{aligned} \Delta &= 0.661 \\ \log a &= 9.91017 \\ \log z_o &= 1.67419 \\ x_m &= 8.7630 \\ \log X_{om} &= 0.87273 \\ \log X_{1m} &= 0.59873 \end{aligned}$$

By equations (*a*) to (*h*), inclusive, we find, the web thickness remaining as before,

$$f = 1686.6$$
$$\log V_1^2 = 6.95993$$
$$\log X'_o = 0.76472 \quad \therefore\ x' = 4.4201 \text{ and } u' = 208.75 \text{ in.}$$

The equations for velocity and pressure are

From $u = 0$ to $u' = 208.75$ in.:

$$v^2 = 6.00247\, X_1 \left\{ 1 + [8.73812-10]\, X_o - [5.98664 - 10]\, X_o^2 \right\}$$
$$p = 4.63188\, X_3 \left\{ 1 + [8.73812-10]\, X_4 - [5.98664 - 10]\, X_5 \right\}$$

From $u' = 208.75$ in. to muzzle:

$$v^2 = [6.27775]\, X_1 \left\{ 1 - [8.71576 - 10]\, X_o \right\}$$
$$p = [4.90716]\, X_3 \left\{ 1 - [8.71576 - 10]\, X_4 \right\}$$

From these equations we find,

Maximum pressure = 37851 lbs. per in.^{2}
Muzzle velocity = 2146 f. s.
Muzzle pressure = 8789 lbs. per in.^{2}

$$v' = 1820.3 \text{ f. s.}$$
$$p' = \left.\begin{matrix} 26299 \\ 18266 \end{matrix}\right\} \text{lbs. per in.}^2$$

The dimensions of these grains are found from the ratios given above, and are as follows:

Diameter of perforations = l_o = 0.0727 in.
Diameter of grain = $11\, l_o$ = 0.8 in.
Length of grain = $200\, l_o$ = 14.54 in.

The following table gives the pressures (p') at different points of the bore for a charge of 314 lbs. of service powder plus an igniter of 9 lbs., making $\tilde{\omega} = 123$ lbs., and also the pressures (p'') of the same charge made up of the grains whose dimensions are given above.

It will be seen from this table, and the previous calculations, that increasing the length of the powder grains relieves the maximum pressure more than is accomplished by lengthening the powder chamber, for the same muzzle energy:

x	u Inches.	p' lbs. per in.[2]	p'' lbs. per in.[2]	$p' - p''$	Remarks.
0.1	4.72	24325	22388	1937	
0.2	9.45	31329	29138	2191	
0.3	14.17	35084	32888	2196	
0.4	18.89	37234	35131	2103	
0.5	23.61	38453	36483	1970	
0.6	28.34	39090	37275	1815	
0.7	33.06	39351	37691	1660	
0.8	37.78	39351	37851	1500	Maximum pressure.
0.9	42.50	39181	37833	1348	
1.0	47.23	38892	37690	1202	
1.5	70.84	36650	36065	585	
2.0	94.45	34150	34019	131	
2.5	118.07	31849	32060	− 211	
3.0	141.68	29820	30293	− 473	
3.5	165.29	28049	28726	− 677	
4.0	188.91	26500	27341	− 841	
4.4201	208.75	25344	26299	− 955	The web thickness is burned at this point.
5.0	236.13	16308	16295	+ 13	
6.0	283.36	13566	13585	− 19	
7.0	330.59	11451	11496	− 45	
8.0	377.82	9775	9838	− 63	
8.763	413.85	8714	8789	− 75	Muzzle.

CHAPTER VI

ON THE RIFLING OF CANNON

Advantages of Rifling.—The greater efficiency of oblong over spherical projectiles is twofold. In the first place they have greater ballistic efficiency,—that is, for the same caliber, muzzle velocity and range, an oblong projectile has a higher average velocity during its flight than a spherical projectile. This gives to the former a flatter trajectory which increases the probability of hitting the target. Experimental firing has demonstrated that the mean deviation of the shots from a rifled gun at medium ranges, when all known and controllable causes of deviation have been eliminated, is only one-third that from a smooth bore. This advantage results both from the greater sectional density of the oblong projectile whereby it is enabled the better to overcome the resistance of the air, and also because this resistance is diminished by the more pointed head.

In the second place the penetration of oblong projectiles, other things being equal, is much greater than can be realized with spherical shot, while the bursting charge of oblong shells is as great or even greater than that of spherical shells on account of their greater length. These are very substantial advantages; but to secure them it is essential that the oblong projectile should keep point foremost in its flight, otherwise it would have neither range, accuracy nor penetration, but would waste its energy beating the air.

The only way to secure steadiness of flight to an oblong projectile is to keep its geometrical axis in the tangent to the trajectory it describes by giving it a high rotary velocity about this axis. This is accomplished by rifling, as it is called,—that

is, by cutting spiral grooves in the surface of the bore into which a projecting copper band, securely encircling the projectile near its base, is forced as soon as motion of translation begins, thus giving to the projectile a rotary motion in addition to its translation as it moves down the bore. The rifling may be such that the grooves (or rifles) have a constant pitch, that is, make a constant angle with the axis of the bore; or, this angle may increase. In the first case the gun is said to be rifled with a constant twist, and in the second case with an increasing twist. In all cases the twist at any point of the bore is measured by the linear distance the projectile would advance while making one revolution supposing the twist at that point to remain constant. This linear distance is always expressed in calibers, and is therefore independent of the unit of length employed.

The Developed Groove. Uniform Twist.—The element of a groove of uniform twist developed upon a plane is evidently a right line $A\,C$ making, with the longitudinal element of the surface of the bore $A\,B$, the constant angle $B\,A\,C$, whose tangent is $B\,C/A\,B$.* Suppose $A\,B$ to be the longitudinal element passing through the beginning of the groove at A, which is near the base of the projectile when in its firing seat and directly in front of the rotating band. Make $A\,B = n\,d$, n being the number of calibers the projectile travels while making one revolution. Then $B\,C$ will be equal to the circumference of the projectile; or, $B\,C = \pi\,d$. If we designate the angle of inclination of the groove, $B\,A\,C$ by β, we shall have

$$\tan\beta = \frac{B\,C}{A\,B} = \frac{\pi\,d}{n\,d} = \frac{\pi}{n} \quad . \quad . \quad . \quad . \quad (1)$$

Increasing Twist.—With a uniform twist the maximum pressure produced on the lands (or sides of the grooves) occurs (as will be shown presently) at the point of maximum pressure

* The simple diagrams required in this Chapter can easily be constructed by the reader.

on the base of the projectile, which point, as we know, is near the beginning of motion. From this point the pressure on the lands decreases to the muzzle where it is not generally more than one-fourth of its maximum value. It is considered by gun-designers a desideratum to have the pressure on the lands as uniform as possible, and to this end recourse is had to an increasing twist—that is, the angle which a groove makes with the axis of the bore, instead of being constant as with a uniform twist, increases from the beginning of rifling toward the muzzle. If this variable angle be represented by θ, we shall still have, as before,

$$\tan \theta = \frac{\pi}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which n is now a variable, decreasing in value as the distance from the origin of rifling increases. If, as before, we take the origin of rectangular coördinates at A, the beginning of rifling, and suppose $A\,B$ to be a longitudinal element of the bore and $B\,C$ the length of arc revolved through by a point on the surface of the projectile while it travels from A to B, then the developed groove, $A\,C$, is a curve convex toward $A\,B$, the axis of abscissas, for the reason that by definition $\tan \theta$ increases from A toward B. The two forms of increasing twist that have been most generally adopted are the parabolic and circular.

General Expression for Pressure on the Lands.—Before attempting to decide upon the best system of rifling it will be necessary to deduce an expression for the pressure upon the lands. Take a cross-section of the bore and suppose for simplicity that there are only two grooves opposite to each other, and let the prolongation of the bearing surface pass through the axis of the bore, as is practically realized in the latest systems. Let M represent the point of application of the bearing surface of the upper groove. We will take three coördinate axes: one axis (x) is coincident with the axis of the bore, while the others (y and z) are in the plane of the cross-section of the bore and

perpendicular to each other. Let the axis y be directed along $O\,M$, and z in a direction perpendicular to $O\,M$.

The pressure at M, whatever may be its direction, can be replaced by the three following mutually perpendicular components: The first, perpendicular to the axis of the bore and consequently acting along the radius through M; the second lying in the plane tangent to the surface of the bore (normal to the radius $O\,M$) and acting along the normal to the groove; the third lying in this same plane and tangent to the groove.

The first of these components will be destroyed by the similar component of the opposite groove and does not enter into the equation of motion of the projectile. The second component, which is the normal pressure against the bearing surface of the groove, we will designate by R. The third component, being in the tangent to the groove, represents the friction on the guiding side of the groove, and may be designated by $f\,R$, in which f is the coefficient of friction. The forces R and $f\,R$ give the following components along the axes x and z:

	Axis of x.	Axis of z.
Force R . .	$-R \sin\theta$	$R\cos\theta$
Force $f\,R$.	$-f\,R\cos\theta$	$-f\,R\sin\theta$

The positive direction of the axis of x is toward the muzzle; that of z in the direction of the force R, and θ is the angle which the groove makes with the axis of x. The full component for the upper groove is:

$$\text{On axis of } x \;.\;\;.\;\; -\;R\,(\sin\theta + f\,R\cos\theta)$$
$$\text{On axis of } z \;.\;\;.\;\; R\,(\cos\theta - f\,R\sin\theta)$$

For the lower groove the component along the axis of x has the same value and sign as the upper one; while the component along the axis of z has the same value but the opposite sign. Besides these forces the projectile is also subjected to the variable

pressure of the powder gases on its base acting along the axis of x in the positive direction, which force call P. If we replace the grooves by the forces enumerated above, we may consider the projectile a free body and apply to it Euler's equations. These equations are six in number; but, as is readily seen, they reduce to two in the problem under consideration, namely: an equation of translation along the axis of x, and of rotation about the same axis, or, what is the same thing, the axis of the projectile. The first equation is

$$M \frac{d^2 x}{d t^2} = P - 2 R (\sin \theta + f \cos \theta) \quad . \quad . \quad . \quad (2)$$

and the second

$$M k^2 \frac{d \omega}{d t} = 2 r R (\cos \theta - f \sin \theta) \quad . \quad . \quad . \quad (3)$$

in which r is the radius of the projectile; ω its angular velocity about its axis and k its radius of gyration.

The angular velocity ω of a projectile about its geometrical axis for an increasing twist, continually increases as it moves along the bore from zero to its muzzle value, which is $\pi v/n r$, v being the muzzle velocity of translation and r the radius of the projectile. Its magnitude at any instant is given by the equation

$$\omega = \frac{d \varphi}{d t}$$

where φ is the angle turned through from the beginning of motion expressed in radians. The angular acceleration is found by differentiating this equation with respect to the time, which gives

$$\text{Angular acceleration} = \frac{d \omega}{d t} = \frac{d^2 \varphi}{d t^2} \quad . \quad . \quad . \quad (4)$$

If we now take x and y as the rectangular coördinates of the developed groove with the origin at the beginning of rifling and the axis of abscissas parallel to the axis of the bore, then x will

represent at any instant the distance travelled by the shot, and the corresponding value of y will be $r\varphi$, where r is the radius of the projectile. Substituting y for φ in (4), gives

$$\frac{d\,\omega}{d\,t} = \frac{1}{r}\frac{d^2 y}{d\,t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting this value of $d\,\omega/d\,t$ in (3), it becomes, putting $k^2/r^2 = \mu$,

$$M\,\mu\frac{d^2 y}{d\,t^2} = 2\,R\,(\cos\theta - f\sin\theta) \quad . \quad . \quad . \quad . \quad (6)$$

Before these equations can be used for determining $2\,R$ we must eliminate $d\,t$; and this we can do by means of the equation of the developed groove. Let

$$y = f\,(x)$$

be this equation. Then employing the usual notation we have

$$\frac{d\,y}{d\,x} = f'\,(x) = \tan\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\frac{d^2 y}{d\,x^2} = f''\,(x)$$

Also, since

$$\frac{d\,y}{d\,t} = \frac{d\,y}{d\,x}\cdot\frac{d\,x}{d\,t} = f'(x)\,\frac{d\,x}{d\,t},$$

we have, by differentiating,

$$\frac{d^2 y}{d\,t^2} = f''\,(x)\left(\frac{d\,x}{d\,t}\right)^2 + f'(x)\frac{d^2 x}{d\,t^2} = v^2 f''\,(x) + \frac{d^2 x}{d\,t^2}\tan\theta$$

Substituting this value of $\frac{d^2 y}{d\,t^2}$, and also the value of $\frac{d^2 x}{d\,t^2}$ from (2) in (6), and solving for $2\,R$, we have

$$2\,R = \frac{\mu\sec\theta\left\{P\,\tan\theta + M\,v^2 f''(x)\right\}}{1 - \tan\theta\left\{f - \mu\,(f + \tan\theta)\right\}} \quad . \quad . \quad . \quad (8)$$

In using this equation $f''\,(x)$ and $\tan\theta$ are obtained from the equation of the developed groove, μ and M from the projectile.

V and P from the equations for velocity and pressure deduced in Chapter IV, while f is determined by experiment. The resulting value of $2R$ will be sum of the rotation pressures on all the lands.

Uniform Twist.—For uniform twist $f''(x)$ is zero and θ becomes constant and equal to β. Its value is given by (1). Making these substitutions the expression for $2R$ becomes for uniform twist

$$2R = \frac{\mu P \tan\beta \sec\beta}{1 - f\tan\beta + \mu\tan\beta\,(f + \tan\beta)} \quad . \quad . \quad (9)$$

In the second member of (9), P, the pressure on the base of the projectile, is the only variable; and therefore $2R$ is directly proportional to this pressure, and is a maximum when P is a maximum. In equation (8) there are four variables in the second member, namely, P, v, θ and $f''(x)$; and it is not obvious on simple inspection where the point of maximum rotation is located. It will be shown, however, by examples that for an increasing twist this point is much nearer the muzzle than when the twist is uniform.

Increasing Twist. Semi-Cubical Parabola.—To continue an increasing twist quite up to the muzzle must conduce to inaccuracy of flight, and especially so when the projectile has partially left the bore so that it has lost its centering. To remedy this the acceleration of rotation near the muzzle is made either zero or constant (preferably the former), in order to relieve the rotating band as much as possible from pressure and to reduce the torsional effect upon the gun, far removed from its support at the trunnions. In all of our sea-coast guns the final twist is made constant beginning at about 2½ calibers from the muzzle. The developed groove for the increasing twist is a semi-cubical parabola whose general equation is

$$y + b = p\,(x + a)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The axis of x is parallel to the axis of the bore and the origin is at the beginning of the rifling, just in front of the rotating band of the projectile when in its firing seat. The coördinates of the vertex of the parabola are $-a$ and $-b$, and these with the parameter p are determined by the particular twist adopted for the beginning and ending of the increasing twist. Suppose the rifling to start with a twist of one turn in n_1 calibers, and that at 2½ calibers from the muzzle where the rifling begins to be constant it has a twist of one turn in n_2 calibers ($n_1 > n_2$). For these two points we have, by (1),

$$\tan\theta_1 = \frac{\pi}{n_1} \text{ and } \tan\theta_2 = \frac{\pi}{n_2}$$

θ_1 and θ_2 being the inclinations of the grooves with the axis of x at the points considered. Differentiating (10), we have,

$$\frac{dy}{dx} = \tan\theta = \frac{3}{2}p(x+a)^{\frac{1}{2}} \quad . \quad . \quad . \quad (11)$$

At the origin $x = 0$, which gives

$$\tan\theta_1 = \frac{3p\sqrt{a}}{2} = \frac{\pi}{n_1}$$

At 2½ calibers from the muzzle where the increasing twist ends, $x = u_2$, and we have at this point

$$\tan\theta_2 = \frac{3p\sqrt{u_2+a}}{2} = \frac{\pi}{n_2}$$

From these two last equations we find

$$a = \frac{u_2}{\left(\frac{n_1}{n_2}\right)^2 - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and

$$p = \frac{2\pi}{3n_1\sqrt{a}} = \frac{2\pi}{3n_2\sqrt{u_2+a}} \quad . \quad . \quad . \quad . \quad (13)$$

Since at the origin x and y are both zero, we find from (10) and (13),

$$b = \frac{2\pi a}{3 n_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Thus all the constants in the equation of the developed groove are given in terms of n_1, n_2 and u_2. Lastly, differentiating (11) gives

$$f''(x) = \frac{3p}{4\sqrt{x+a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

If the vertex of the semi-cubical parabola is at the origin, or beginning of rifling, a and b are zero, and (10) becomes

$$y = p x^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

In this case the twist is zero at the origin and increases to one turn in n_2 calibers near the muzzle. The values of $\tan\theta$, p and $f''(x)$ for this particular form of rifling are deduced from (11), (13), and (15), by making a zero. This form of groove is that adopted by the navy for all their heavy guns of recent construction.

Common parabola.—The equation of the common parabola is

$$y + b = p(x+a)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where $-a$ and $-b$ are the coördinates of the vertex. The constants are determined as for the semi-cubical parabola, and are as follows:

$$a = \frac{u_2}{\left(\frac{n_1}{n_2}\right) - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$b = \frac{\pi a}{2 n_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$p = \frac{\pi}{2 a n_1} = \frac{\pi}{2 n_2 (u_2 + a)} \quad . \quad . \quad . \quad . \quad (20)$$

$$f''(x) = 2p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Relative Width of Grooves and Lands.—In our service siege and sea-coast guns the number, N, of grooves (or lands) is given by the equation

$$N = 6\,d$$

in which d is the diameter of the bore in inches, and is a whole number for each of these guns. If w_g is the width of a groove and w_l the width of a land, we have the relation

$$w_g + w_l = \frac{\pi\,d}{6\,d} = \frac{\pi}{6} = 0.5236 \text{ inches.}$$

The best authorities lay down the rule that the width of a groove should be at least double that of a land. In our guns the lands are made 0.15 in. wide, and the grooves are therefore $0.5236 - 0.15 = 0.3736$ in. wide.

Application to the 10-inch B. L. R. Model 1888.—This gun has 60 grooves which, beginning at 20.1 inches from the bottom of the bore with a twist of one turn in 50 calibers, increase to one in 25 at 20 inches from the muzzle, and from thence continue uniform. We therefore have $n_1 = 50$ and $n_2 = 25$. The bore is 22.925 ft. long, and therefore $u_2 = 19.583$ ft. The developed groove is a semi-cubical parabola whose equation is (10). The constants are

$$a = \frac{19.583}{3} = 6.528 \text{ ft.}$$

$$b = \frac{6.528\,\pi}{75} = 0.27344 \text{ ft.}$$

$$p = \frac{2\,\pi}{150\sqrt{6.528}} = 0.016395.$$

The equation of the developed groove (changing x to u to indicate travel of projectile) is therefore,

$$y + 0.27344 = 0.16395\,(u + 6.528)^{\frac{3}{2}} \quad . \quad . \quad (22)$$

in which y will be given in feet.

From (11), we have

$$\tan\theta = 0.024592\sqrt{u + 6.528}$$

Making u zero in this last equation gives

$$\theta_1 = 3^\circ\ 35'\ 42''$$

which is the inclination at which the groove starts. At 20 inches from the muzzle where the twist becomes uniform (and which is therefore a point of discontinuity on the developed groove) we have $u_2 = 19.583$; and at this point

$$\theta_2 = 7^\circ\ 9'\ 45''$$

This value of θ is retained to the muzzle.

From (15), we have

$$f''(x) = \frac{0.012296}{\sqrt{u + 6.528}}$$

This function decreases from the origin to the point of discontinuity. From this point to the muzzle $f''(x)$ is zero.

If we put

$$K = \frac{\mu \sec\theta}{1 - \tan\theta\left\{f - \mu(f + \tan\theta)\right\}} \quad . \quad . \quad . \quad (23)$$

equation (8) becomes

$$2R = K\left\{P\tan\theta + Mv^2 f''(x)\right\} \quad . \quad . \quad (24)$$

Captain (now Sir Andrew) Noble, as the result of very careful experiments made by him with 12-cm. quick-firing guns, found $f = 0.2$, and this value will be adopted in what follows. We also have for cored shot $\mu = 0.5$, nearly. Substituting these values of f and μ in (23), it will be found that K increases very slowly as θ increases. The values of K for $u = 0$ and $u = 19.583$ are, respectively, 0.5032 and 0.5064. We might therefore take for K the arithmetical mean of these two values and write (24)

$$2R = 0.5048\left\{P\tan\theta + Mv^2 f''(x)\right\} \quad . \quad . \quad (25)$$

without any material error. This formula may be employed for all our sea-coast guns.

If the 10-inch gun were rifled with the kind of groove given by (16), we should find

$$p = \frac{2\pi}{75\sqrt{19.583}} = 0.018931$$

$$\tan\theta = 0.028397\sqrt{u}$$

$$f''(x) = \frac{0.014198}{\sqrt{u}}$$

In this form of rifling the initial inclination of the grooves is zero and increases to $7^\circ\ 9'\ 45''$ at 20 inches from the muzzle, where the twist becomes uniform. Between this point and the muzzle, $f''(x)$ is zero.

Uniform Twist.—If we suppose the 10-inch gun to be rifled throughout with a uniform twist of one turn in 25 calibers, we have $\beta = 7^\circ\ 9'\ 45''$. Employing the values of μ and f already given, (9) reduces to

$$2R = 0.063624\,P. \qquad (26)$$

Working Expressions.—If the equation of the developed groove is (10), we have

$$2R = K\tan\theta\left\{P + \frac{Mv^2}{2(u+a)}\right\}. \qquad (27)$$

and

$$\tan\theta = \frac{\pi}{25}\left(\frac{u+a}{u_2+a}\right)^{\frac{1}{2}}$$

If (16) is the equation of the developed groove, we have

$$2R = K\tan\theta\left\{P + \frac{Mv^2}{2u}\right\} \qquad (28)$$

and

$$\tan\theta = \frac{\pi}{25}\left(\frac{u}{u_2}\right)^{\frac{1}{2}}$$

Pressure on the Lands of the 10-Inch B. L. R.—The equations for velocity and pressure for this gun are the following:

$$v^2 = [6.20536]\, X_1 \left\{ 1 - [8.59381 - 10]\, X_o \right\} \quad . \quad . \quad (29)$$

and

$$p = [4.72060]\, X_3 \left\{ 1 - [8.59381 - 10]\, X_4 \right\} \quad . \quad . \quad (30)$$

The gun and firing data were $V_c = 7064$ c. i., $u_m = 22.925$ ft., $\tilde{\omega} = 250$ lbs. of brown cocoa powder, $w = 575$ lbs., muzzle velocity 1975 f. s., maximum pressure on base of projectile, 33300 lbs. per in.^{2}, $\Delta = 0.98$, and $z_o = 3.461$ ft. It will be convenient to change (30) so that it will give the entire pressure (P) on the base of the projectile; and to avoid large numbers we will adopt the ton as the unit of weight. We then have

$$P = \frac{\pi d^2 p}{8960}$$

and (30) becomes

$$P = [3.26544]\, X_3 \left\{ 1 - [8.59381 - 10]\, X_4 \right\} . \quad . \quad (30')$$

Finally, the mass of the projectile expressed in tons is

$$M = \frac{575}{2240\, g} = [7.90210 - 10]$$

We have now all the formulas and data necessary for computing the pressures on the lands of the 10-inch B. L. R., by means of (26), (27), and (28), for the three principal systems of rifling adopted in our service. These calculations are given in the table on page 183.

The last three columns of this table show that the maximum pressure on the lands is greater for uniform twist than for either form of increasing twist; but the difference between these maxima is not very great. Moreover, the maximum pressure for uniform twist occurs at the trunnions where its torsional effect upon the gun—so far as deranging the aim is concerned—is a

Pressures on lands required to produce rotation of shot in the 10-inch B. L. R. for different systems of rifling. Charge 250 lbs. Projectile 575 lbs. Muzzle velocity 1975 f. s. Maximum pressure on base of projectile 33300 lbs. per square inch.

x	Travel of Shot, feet	Velocity of Shot, f. s.	Pressure on Base of Shot, tons	Pressure on Lands. Tons		
				Uniform Twist	Increasing Twist Eq. (28)	Increasing Twist, Eq. (27)
0.0	0.	0.0	0.0	0.0	0.0	0.0
.1	0.3461	227.7	841.1	53.5	12.0	27.0
.2	0.6922	366.7	1036.4	65.9	21.5	36.9
0.3	1.0382	478.1	1122.4	71.4	29.0	42.3
.4	1.3843	572.4	1158.3	73.7	35.3	46.1
.5	1.7304	654.7	1167.4	74.3	40.9	48.9
0.6	2.0765	727.8	1161.1	73.9	44.8	51.1
.7	2.4226	793.6	1145.6	72.9	48.5	52.9
.8	2.7686	853.4	1124.7	71.6	51.6	54.3
0.9	3.1147	908.2	1100.7	70.0	54.3	55.5
1.0	3.4608	958.7	1075.0	68.4	56.7	56.5
1.1	3.8069	1005.6	1048.5	66.7	58.7	57.3
1.2	4.1530	1049.3	1021.9	65.0	60.5	58.1
1.3	4.4991	1090.2	995.5	63.3	62.1	58.7
1.4	4.8452	1128.6	969.7	61.7	63.5	59.2
1.5	5.1912	1164.7	944.5	60.1	64.7	59.7
1.6	5.5372	1198.9	920.1	58.5	65.8	60.1
1.7	5.8833	1231.4	896.4	57.0	66.7	60.5
1.8	6.2294	1262.1	873.6	55.6	67.5	60.8
1.9	6.5755	1291.4	851.7	54.2	68.2	61.1
2.0	6.9216	1319.4	830.5	52.8	69.0	61.3
3.0	10.3824	1543.4	659.4	42.0	72.7	62.4
4.0	13.8432	1702.8	541.4	34.4	73.5	62.3
5.0	17.3040	1824.9	456.4	29.0	73.2	61.6
5.6586	19.5833	1891.6	412.4	26.2	72.6	61.0
6.0000	20.7648	1922.8	392.4	25.0	25.0	25.0
6.6242	22.9250	1975.0	359.9	22.9	22.9	22.9

minimum; while the position of the maximum pressure upon the lands for either form of increasing twist is well down the chase. It is difficult to see any superiority of an increasing twist over a uniform twist, especially in view of the fact demonstrated by Captain Noble's experiments, that the energy expended in giving rotation to the projectile with rifling having an increasing twist is nearly twice as great as with a uniform twist.

Application to the 14-inch Rifle.—This gun has 126 grooves and the same number of lands, in this respect differing from the rule followed with the other seacoast guns. The values of n_1, n_2, θ_1 and θ_2 are the same as those found for the 10-inch rifle. The rifling begins 7.05 inches from the base of the projectile when in its firing seat and becomes uniform 22.8 inches from the muzzle. Therefore

$$u_2 = 413.85 - (7.05 + 22.8) = 384 \text{ inches.}$$

From (12), (13), and (14), we now find

$$a = 128$$
$$p = 0.0037024$$
$$b = 5.36165$$

Therefore the equation of the developed groove is

$$y + 5.36165 = 0.0037024\,(u + 128)^{\frac{3}{2}}$$

From (11) and (15), we have, finally,

$$\tan\theta = 0.0055536\sqrt{u + 128}$$

$$f''(x) = \frac{0.0027768}{\sqrt{u+128}}$$

$$M = \frac{1660}{2240\,g} = 0.02304$$

and

$$P = \frac{\pi\, d^2\, p}{8960} = 0.06872\, p$$

in which P is the entire pressure on the base of the projectile in

tons while p is the pressure in pounds per square inch given by the equation on page 166, for a charge of 314 lbs.

Substituting these expressions for tan $\theta, f''(x)$, and M in (25) and reducing, we have the working expression

$$2R = [7.44769-10]\sqrt{u+128}\left\{P+[8.06145-10]\frac{v^2}{u+128}\right\}$$

in which $2R$ is the normal pressure on all the lands in tons and u the travel of the projectile in inches. To determine the normal pressure on each land $2R$ must be divided by 126.

For a uniform twist $2R$ is given by (26).

In the following table v and p were computed by the formulas on page 166 for a charge of 314 lbs., and P and $2R$ by the formulas given above:

x	u inches.	v f. s.	P tons.	$2R$ Increasing T	$2R$ Uniform T
0.1	4.72	198.7	1671.7	54.1	106.4
0.2	9.45	325.2	2153.0	71.1	137.0
0.3	14.17	429.1	2411.1	81.1	153.4
0.4	18.89	518.9	2558.8	87.7	162.8
0.5	23.61	598.5	2642.6	92.2	168.2
0.6	28.34	670.3	2686.3	95.3	171.3
0.7	33.06	735.8	2704.3	97.6	172.1
0.8	37.78	796.1	2704.3	99.2	172.1
0.9	42.50	852.1	2692.6	100.4	171.3
1.0	47.23	904.3	2672.8	101.2	170.1
1.5	70.84	1123.3	2518.7	102.5	160.3
2.0	94.45	1295.1	2346.9	101.8	149.3
2.5	118.07	1436.7	2188.7	100.5	139.3
3.0	141.68	1557.5	2049.3	99.1	130.4
3.5	165.29	1662.9	1927.6	97.8	122.7
4.0	188.91	1756.5	1821.1	96.5	115.9
5.0	236.13	1891.0	1120.7	66.0	71.3
6.0	283.36	1981.2	932.3	59.3	59.3
7.0	330.59	2053.7	787.0	53.6	50.1
8.0	377.82	2113.3	671.8	48.8	42.7
8.763	413.85 Muzzle	2152.0	598.8		

Influence of the Rifling for a Uniform Twist.—For a uniform twist we have

$$\omega = \frac{\pi v}{n r}*$$

where n is constant, and r is the radius of the projectile. Differentiating with respect to t we have

$$\frac{d\,\omega}{d\,t} = \frac{\pi}{n r}\,\frac{d\,v}{d\,t} = \frac{\pi}{n r}\,\frac{d^2 x}{d\,t^2}$$

Substituting this value of $d\,\omega / d\,t$ in (3), it becomes

$$\frac{\pi\,\mu\,M}{n}\,\frac{d^2 x}{d\,t^2} = 2\,R\,(\cos\beta - f\sin\beta) \quad . \quad . \quad . \quad (31)$$

Eliminating $2\,R$ between equations (2) and (31), gives

$$P = \frac{d^2 x}{d\,t^2}\left\{M + \frac{\pi\,\mu\,M}{n}\,\frac{f + \tan\beta}{1 - f\tan\beta}\right\} \quad . \quad . \quad . \quad (32)$$

If the bore were smooth the equation of translation of the projectile would be

$$M\frac{d^2 x}{d\,t^2} - P;$$

from which it appears that the effect of the grooves upon the motion of the projectile for a constant twist is equivalent to increasing the mass of the projectile by the quantity

$$\frac{\pi\,\mu\,M}{n} \cdot \frac{f + \tan\beta}{1 - \tan\beta}$$

By making $f = 0.2$, $n = 25$, $\mu = 0.5$ and $\beta = 7^\circ\,9'\,45''$, the value of this supplemental term is found to be $0.021\,M$. That is, the retarding effect of a constant twist of one turn in 25 calibers is equivalent to increasing the weight of the projectile 2 per cent.

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* Artillery Circular, N, p. 201.

TABLES

TABLE I

Logarithms of the functions X_0, X_1, X_2, X_3, X_4, X_5, with differences for interpolation.

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
0.01	9.53911	34760	7.05911	103529	7.52000	68769	9.23296	32763	9.66437	34885	9.30059	69719
0.05	9.88671	14823	8.09440	43569	8.20769	28746	9.56059	12434	0.01322	14973	9.99778	29885
0.10	0.03494	8584	8.53009	24888	8.49515	16304	9.68493	6305	0.16295	9728	0.29663	17397
0.15	0.12078	6033	8.77897	17273	8.65819	11240	9.74798	3855	0.25023	6171	0.47060	12287
0.20	0.18111	4639	8.95170	13121	8.77059	8482	9.78653	2553	0.31194	4771	0.59347	9487
0.25	0.22750	3759	9.08291	10511	8.85541	6752	9.81206	1756	0.35965	3886	0.68834	7718
0.30	0.26509	3152	9.18802	8720	8.92293	5568	9.82962	1229	0.39851	3276	0.76552	6500
0.35	0.29661	2711	9.27522	7420	9.87861	4709	9.84191	860	0.43127	2829	0.83052	5608
0.40	0.32372	498	9.34942	1357	9.02570	859	9.85051	136	0.45956	522	0.88660	1033
0.41	0.32870	486	9.36299	1320	9.03429	834	9.85187	126	0.46478	509	0.89693	1008
0.42	0.33356	475	9.37619	1285	9.04263	811	8.95313	117	0.46987	497	0.90701	984
0.43	0.33831	463	9.38904	1252	9.05074	789	9.85430	109	0.47484	485	0.91685	962
0.44	0.34294	452	9.40156	1219	9.05863	767	9.85539	101	0.47969	475	0.92647	940
0.45	0.34746	440	9.41375	1188	9.06630	746	9.85640	92	0.48444	463	0.93587	918
0.46	0.35186	431	9.42563	1159	9.07376	727	9.85732	84	0.48907	453	0.94505	898
0.47	0.35617	422	9.43722	1131	9.08103	709	9.85816	78	0.49360	444	0.95403	878
0.48	0.36039	413	9.44853	1104	9.08812	693	9.85894	71	0.49804	434	0.96281	859
0.49	0.36452	403	9.45957	1079	9.09505	676	9.85965	63	0.50238	425	0.97140	840
0.50	0.36855	394	9.47036	1053	9.10181	659	9.86028	57	0.50663	416	0.97980	822
0.51	0.37249	387	9.48089	1030	9.10840	643	9.86085	52	0.51079	408	0.98802	806
0.52	0.37636	379	9.49119	1007	9.11483	628	9.86137	46	0.51487	400	0.99608	791
0.53	0.38015	371	9.50126	986	9.12111	615	9.86183	41	0.51887	392	1.00399	776
0.54	0.38386	364	9.51112	965	9.12726	601	9.86224	36	0.52279	386	1.01175	762
0.55	0.38750	356	9.52077	944	9.13327	587	9.86260	31	0.52665	378	1.01937	747
0.56	0.39106	350	9.53021	925	9.13914	575	9.86291	26	0.53043	371	1.02684	733

TABLE I—*Continued*

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
0.57	0.39456	344	9.53946	906	9.14489	563	9.86317	22	0.53414	364	1.03417	720
0.58	0.39800	338	9.54852	888	9.15052	550	9.86339	18	0.53778	358	1.04137	707
0.59	0.40138	331	9.55740	870	9.15602	539	9.86357	14	0.54136	352	1.04844	695
0.60	0.40469	325	9.56610	853	9.16141	528	9.86371	10	0.54488	346	1.05539	682
0.61	0.40794	320	9.57463	837	9.16669	518	9.86381	6	0.54834	340	1.06221	671
0.62	0.41114	314	9.58300	821	9.17187	507	9.86387	+3	0.55174	335	1.06892	660
0.63	0.41428	309	9.59121	806	9.17694	497	9.86390	0	0.55509	329	1.07552	649
0.64	0.41737	304	9.59927	792	9.18191	487	9.86390	−4	0.55838	323	1.08201	639
0.65	0.42041	299	9.60719	777	9.18678	477	9.86386	7	0.56161	318	1.08840	628
0.66	0.42340	294	9.61496	763	9.19155	469	9.86379	9	0.56479	313	1.09468	619
0.67	0.42634	290	9.62259	750	9.19624	461	9.86370	12	0.56792	309	1.10087	609
0.68	0.42924	285	9.63009	737	9.20085	453	9.86358	15	0.57101	304	1.10696	600
0.69	0.43209	280	9.63746	725	9.20538	444	9.86343	18	0.57405	300	1.11296	591
0.70	0.43489	276	9.64471	712	9.20982	436	9.86325	20	0.57705	295	1.11887	583
0.71	0.43765	272	9.65183	701	9.21418	429	9.86305	23	0.58000	291	1.12470	574
0.72	0.44037	268	9.65884	689	9.21847	421	9.86282	25	0.58291	287	1.13044	566
0.73	0.44305	264	9.66573	678	9.22268	414	9.86257	27	0.58578	283	1.13610	557
0.74	0.44569	260	9.67251	667	9.22682	407	9.86230	29	0.58861	279	1.14167	548
0.75	0.44829	256	9.67918	656	9.23089	400	9.86201	31	0.59140	275	1.14715	542
0.76	0.45085	252	9.68574	646	9.23489	393	9.86170	33	0.59415	271	1.15257	535
0.77	0.45337	249	9.69220	636	9.23882	387	9.86137	35	0.59686	268	1.15792	527
0.78	0.45586	246	9.69856	627	9.24269	381	9.86102	36	0.59954	264	1.16319	520
0.79	0.45832	243	9.70483	617	9.24650	375	9.86066	38	0.60218	261	1.16839	513
0.80	0.46075	239	9.71100	608	9.25025	369	9.86028	40	0.60479	257	1.17352	506
0.81	0.46314	236	9.71708	599	9.25394	363	9.85988	42	0.60736	254	1.17858	500

TABLE I—*Continued*

x	$\log X_0$	D	$\log X_1$	D	$\log X_2$	D	$\log X_3$	D	$\log X_4$	D	$\log X_5$	D
0.82	0.46550	233	9.72307	590	9.25757	357	9.85946	−44	0.60990	251	1.18358	494
0.83	0.46783	231	9.72897	582	9.26114	352	9.85902	45	0.61241	248	1.18852	488
0.84	0.47014	227	9.73479	573	9.26466	346	9.85857	46	0.61489	244	1.19340	481
0.85	0.47241	224	9.74052	565	9.26812	341	9.85811	47	0.61733	242	1.19821	476
0.86	0.47465	221	9.74617	557	9.27153	336	9.85764	48	0.61975	239	1.20297	470
0.87	0.47686	219	9.75174	550	9.27489	331	9.85716	50	0.62214	236	1.20767	464
0.88	0.47905	216	9.75724	543	9.27820	326	9.85666	51	0.62450	233	1.21231	459
0.89	0.48121	213	9.76267	535	9.28146	322	9.85615	53	0.62683	230	1.21690	453
0.90	0.48334	211	9.76802	528	9.28468	317	9.85562	54	0.62913	228	1.22143	448
0.91	0.48545	208	9.77330	521	9.28785	313	9.85508	55	0.63141	225	1.22591	443
0.92	0.48753	206	9.77851	514	9.29098	308	9.85453	56	0.63366	223	1.23034	438
0.93	0.48959	203	9.78365	507	9.29406	304	9.85397	56	0.63589	220	1.23472	433
0.94	0.49162	201	9.78872	501	9.29710	300	9.85341	57	0.63809	218	1.23905	427
0.95	0.49363	198	9.79373	494	9.30010	296	9.85284	58	0.64027	215	1.24332	423
0.96	0.49561	196	9.79867	488	9.30306	292	9.85226	59	0.64242	213	1.24755	419
0.97	0.49757	194	9.80355	482	9.30598	288	9.85167	60	0.64455	211	1.25174	414
0.98	0.49951	193	9.80837	476	9.30886	284	9.85107	61	0.64666	209	1.25588	410
0.99	0.50144	190	9.81313	471	9.31170	280	9.85046	62	0.64875	206	1.25998	406
1.00	0.50334	188	9.81784	465	9.31450	276	9.84984	63	0.65081	204	1.26404	401
1.01	0.50522	186	9.82249	459	9.31726	273	9.84921	63	0.65285	202	1.26805	397
1.02	0.50708	184	9.82708	454	9.31999	270	9.84858	64	0.65487	200	1.27202	393
1.03	0.50892	182	9.83162	448	9.32269	266	9.84794	65	0.65687	198	1.27595	389
1.04	0.51074	181	9.83610	443	9.32535	263	9.84729	65	0.65885	197	1.27984	385
1.05	0.51255	178	9.84053	438	9.32798	260	9.84664	66	0.66082	194	1.28369	381
1.06	0.51433	176	9.84491	433	9.33058	257	9.84598	66	0.66276	192	1.28750	378

TABLE I—*Continued*

x	$\log X_0$	D	$\log X_1$	D	$\log X_2$	D	$\log X_3$	D	$\log X_4$	D	$\log X_5$	D
1.07	0.51609	175	9.84924	428	9.33315	253	9.84532	−67	0.66468	190	1.29128	374
1.08	0.51784	173	9.85352	423	9.33568	250	9.84465	68	0.66658	189	1.29502	370
1.09	0.51957	171	9.85775	418	9.33818	247	9.84397	68	0.66847	187	1.29872	367
1.10	0.52128	170	9.86193	414	9.34065	244	9.84329	69	0.67034	185	1.30239	363
1.11	0.52298	168	9.86607	409	9.34309	241	9.84260	69	0.67219	183	1.30602	359
1.12	0.52466	166	9.87016	405	9.34550	239	9.84191	70	0.67402	182	1.30961	356
1.13	0.52632	165	9.87421	400	9.34789	236	9.84121	70	0.67584	180	1.31317	353
1.14	0.52797	163	9.87821	396	9.35025	233	9.84051	70	0.67764	178	1.31670	350
1.15	0.52960	161	9.88217	392	9.35258	230	9.83981	71	0.67942	176	1.32020	346
1.16	0.53121	160	9.88609	388	9.35488	228	9.83910	71	0.68118	175	1.32366	343
1.17	0.53281	158	9.88997	384	9.35716	225	9.83839	72	0.68293	173	1.32709	340
1.18	0.53439	157	9.89381	380	9.35941	223	9.83767	72	0.68466	172	1.33049	338
1.19	0.53596	156	9.89761	375	9.36164	220	9.83695	72	0.68638	171	1.33387	334
1.20	0.53752	154	9.90136	372	9.36384	218	9.83623	73	0.68809	169	1.33721	331
1.21	0.53906	152	9.90508	368	9.36602	215	9.83550	73	0.68978	168	1.34052	328
1.22	0.54058	151	9.90876	364	9.36817	213	9.83477	73	0.69146	166	1.34380	325
1.23	0.54209	150	9.91240	361	9.37030	211	9.83404	74	0.69312	165	1.34705	323
1.24	0.54359	149	9.91601	357	9.37241	208	9.83330	74	0.69477	163	1.35028	320
1.25	0.54508	148	9.91958	354	9.37449	206	9.83256	74	0.69640	162	1.35348	317
1.26	0.54656	146	9.92312	350	9.37655	204	9.83182	75	0.69802	160	1.35665	315
1.27	0.54802	145	9.92662	347	9.37859	202	9.83107	75	0.69962	159	1.35980	312
1.28	0.54947	144	9.93009	343	9.38061	200	9.83032	75	0.70121	158	1.36292	309
1.29	0.55091	143	9.93352	341	9.38261	198	9.82957	75	0.70279	157	1.36601	307
1.30	0.55234	141	9.93693	336	9.38459	195	9.82882	75	0.70436	156	1.36908	304
1.31	0.55375	140	9.94029	334	9.38654	193	9.82807	76	0.70592	154	1.37212	302

TABLE I—*Continued*

x	$\log X_0$	D	$\log X_1$	D	$\log X_2$	D	$\log X_3$	D	$\log X_4$	D	$\log X_5$	D
1.32	0.55515	139	9.94363	331	9.38847	192	9.82731	−76	0.70746	153	1.37514	300
1.33	0.55654	138	9.94694	328	9.39039	190	9.82655	76	0.70899	152	1.37814	297
1.34	0.55792	137	9.95022	324	9.39229	188	9.82579	76	0.71051	150	1.38111	295
1.35	0.55929	136	9.95346	322	9.39417	186	9.82503	77	0.71201	149	1.38406	293
1.36	0.56065	135	9.95668	319	9.39603	184	9.82426	77	0.71350	148	1.38699	290
1.37	0.56200	134	9.95987	316	9.39787	182	9.82349	77	0.71498	147	1.38989	288
1.38	0.56334	132	9.96303	313	9.39969	181	9.82272	76	0.71645	146	1.39277	286
1.39	0.56466	131	9.96616	310	9.40150	179	9.82196	77	0.71791	145	1.39563	283
1.40	0.56597	130	9.96926	307	9.40329	177	9.82119	78	0.71936	144	1.39846	280
1.41	0.56727	129	9.97233	304	9.40506	175	9.82041	77	0.72080	143	1.40126	279
1.42	0.56856	128	9.97537	302	9.40681	174	9.81964	77	0.72223	142	1.40405	277
1.43	0.56984	127	9.97839	299	9.40855	173	9.81887	77	0.72365	140	1.40682	275
1.44	0.57111	127	9.98138	298	9.41028	170	9.81810	78	0.72505	139	1.40957	273
1.45	0.57238	125	9.98436	295	9.41198	169	9.81732	78	0.72644	139	1.41230	272
1.46	0.57363	124	9.98731	292	9.41367	168	9.81654	78	0.72783	138	1.41502	270
1.47	0.57487	124	9.99023	290	9.41535	166	9.81576	78	0.72921	137	1.41772	268
1.48	0.57611	123	9.99313	287	9.41701	164	9.81498	78	0.73058	136	1.42040	266
1.49	0.57734	122	9.99600	284	9.41865	163	9.81420	77	0.73194	134	1.42306	263
1.50	0.57856	121	9.99884	282	9.42028	161	9.81343	78	0.73328	134	1.42569	261
1.51	0.57977	120	0.00166	280	9.42189	160	9.81265	78	0.73462	133	1.42830	260
1.52	0.58097	119	0.00446	278	9.42349	158	9.81187	78	0.73595	132	1.43090	258
1.53	0.58216	118	0.00724	275	9.42507	157	9.81109	78	0.73727	131	1.43348	256
1.54	0.58334	118	0.00999	273	9.42664	156	9.81031	78	0.73858	130	1.43604	254
1.55	0.58452	116	0.01272	271	9.42820	155	9.80953	78	0.73988	129	1.43858	252
1.56	0.58568	116	0.01543	269	9.42975	153	9.80875	78	0.74117	128	1.44110	251

TABLE I—*Continued*

x	$\log X_0$	D	$\log X_1$	D	$\log X_2$	D	$\log X_3$	D	$\log X_4$	D	$\log X_5$	D
1.57	0.58684	115	0.01812	267	9.43128	152	9.80797	−78	0.74245	127	1.44361	249
1.58	0.58799	114	0.02079	264	9.43280	150	9.80719	79	0.74372	127	1.44610	248
1.59	0.58913	113	0.02343	262	9.43430	149	9.80640	79	0.74499	126	1.44858	246
1.60	0.59026	113	0.02605	260	9.43579	147	9.80561	78	0.74625	125	1.45104	244
1.61	0.59139	112	0.02865	258	9.43726	146	9.80483	78	0.74750	124	1.45348	243
1.62	0.59251	111	0.03123	256	9.43872	145	9.80405	78	0.74874	123	1.45591	241
1.63	0.59362	110	0.03379	255	9.44017	145	9.80327	79	0.74997	123	1.45832	240
1.64	0.59472	110	0.03634	253	9.44162	143	9.80248	79	0.75120	122	1.46072	238
1.65	0.59582	109	0.03887	251	9.44305	142	9.80169	78	0.75242	121	1.46310	237
1.66	0.59691	108	0.04138	249	9.44447	141	9.80091	78	0.75363	120	1.46547	235
1.67	0.59799	107	0.04387	247	9.44588	140	9.80013	78	0.75483	120	1.46782	234
1.68	0.59906	107	0.04634	245	9.44728	138	9.79935	79	0.75603	119	1.47016	232
1.69	0.60013	106	0.04879	243	9.44866	137	9.79856	79	0.75722	118	1.47248	230
1.70	0.60119	105	0.05122	241	9.45003	136	9.79777	78	0.75840	117	1.47478	229
1.71	0.60224	105	0.05363	239	9.45139	135	9.79699	78	0.75957	116	1.47707	227
1.72	0.60329	104	0.05602	238	9.45274	134	9.79621	78	0.76073	116	1.47934	226
1.73	0.60433	103	0.05840	237	9.45408	133	9.79543	78	0.76189	115	1.48160	225
1.74	0.60536	103	0.06077	234	9.45541	131	9.79465	79	0.76304	115	1.48385	223
1.75	0.60639	102	0.06311	233	9.45672	131	9.79386	78	0.76419	114	1.48608	222
1.76	0.60741	101	0.06544	231	9.45803	130	9.79308	78	0.76533	113	1.48830	221
1.77	0.60842	101	0.06775	230	9.45933	129	9.79320	78	0.76646	112	1.49051	219
1.78	0.60943	100	0.07005	228	9.46062	128	9.79152	78	0.76758	112	1.49270	218
1.79	0.61043	100	0.07233	226	9.46190	127	9.79074	78	0.76870	111	1.49488	217
1.80	0.61143	99	0.07459	225	9.46317	126	9.78996	78	0.76981	110	1.49705	216
1.81	0.61242	98	0.07684	223	9.46443	124	9.78918	78	0.77091	109	1.49921	214

TABLE I—*Continued*

x	$\log X_0$	D	$\log X_1$	D	$\log X_2$	D	$\log X_3$	D	$\log X_4$	D	$\log X_5$	D
1.82	0.61340	98	0.07907	222	9.46567	124	9.78840	−78	0.77201	109	1.50135	213
1.83	0.61438	97	0.08129	220	9.46691	122	9.78762	78	0.77310	109	1.50348	212
1.84	0.61535	97	0.08349	218	9.46813	122	9.78684	77	0.77419	108	1.50560	210
1.85	0.61632	96	0.08567	217	9.46935	121	9.78607	78	0.77527	107	1.50770	209
1.86	0.61728	95	0.08784	216	9.47056	120	9.78529	78	0.77634	107	1.50979	208
1.87	0.61823	95	0.09000	214	9.47176	119	9.78451	78	0.77741	106	1.51187	207
1.88	0.61918	94	0.09214	213	9.47295	119	9.78373	77	0.77847	105	1.51394	205
1.89	0.62012	94	0.09427	211	9.47414	118	9.78296	77	0.77952	105	1.51599	204
1.90	0.62106	93	0.09638	210	9.47532	117	9.78219	77	0.78057	104	1.51803	203
1.91	0.62199	93	0.09848	209	9.47649	116	9.78142	77	0.78161	104	1.52006	202
1.92	0.62292	92	0.10057	207	9.47765	115	9.78065	77	0.78265	103	1.52208	201
1.93	0.62384	92	0.10264	206	9.47880	114	9.77988	77	0.78368	103	1.52409	200
1.94	0.62476	91	0.10470	205	9.47994	114	9.77911	78	0.78471	102	1.52609	199
1.95	0.62567	91	0.10675	203	9.48108	113	9.77833	77	0.78573	101	1.52808	198
1.96	0.62658	90	0.10878	202	9.48221	112	9.77756	77	0.78674	101	1.53006	197
1.97	0.62748	90	0.11080	201	9.48333	111	9.77679	77	0.78775	100	1.53203	196
1.98	0.62838	89	0.11281	199	9.48444	110	9.77602	77	0.78875	100	1.53399	195
1.99	0.62927	88	0.11480	198	9.48554	109	9.77525	76	0.78975	100	1.53594	194
2.00	0.63015	176	0.11678	392	9.48663	216	9.77449	153	0.79075	197	1.53788	383
2.02	0.63191	174	0.12070	387	9.48879	214	9.77296	153	0.79272	195	1.54171	379
2.04	0.63365	172	0.12457	382	9.49093	211	9.77143	152	0.79467	193	1.54550	376
2.06	0.63537	170	0.12839	378	9.49304	208	9.76991	152	0.79660	191	1.54926	373
2.08	0.63707	168	0.13217	374	9.49512	205	9.76839	152	0.79851	189	1.55299	369
2.10	0.63875	166	0.13591	369	9.49717	203	9.76687	151	0.80040	187	1.55668	365
2.12	0.64041	165	0.13960	365	9.49920	200	9.76536	151	0.80227	185	1.56033	361

TABLE I—*Continued*

x	$\log X_0$	D	$\log X_1$	D	$\log X_2$	D	$\log X_3$	D	$\log X_4$	D	$\log X_5$	D
2.14	0.64206	163	0.14325	361	9.50120	197	9.76385	−150	0.80412	184	1.56394	358
2.16	0.64369	162	0.14686	357	9.50317	195	9.76235	150	0.80596	182	1.56752	354
2.18	0.64531	160	0.15043	352	9.50512	192	9.76085	150	0.80778	180	1.57106	350
2.20	0.64691	158	0.15395	348	9.50704	190	9.75935	149	0.80958	178	1.57456	347
2.22	0.64849	157	0.15743	344	9.50894	188	9.75786	149	0.81136	177	1.57803	344
2.24	0.65006	155	0.16087	340	9.51082	185	9.75637	148	0.81313	175	1.58147	340
2.26	0.65161	154	0.16427	337	9.51267	183	9.75489	148	0.81488	173	1.58487	337
2.28	0.65315	152	0.16764	333	9.51450	180	9.75341	148	0.81661	172	1.58824	334
2.30	0.65467	151	0.17097	329	9.51630	178	9.75193	147	0.81833	170	1.59158	331
2.32	0.65618	149	0.17426	326	9.51808	176	9.75046	147	0.82003	168	1.59489	328
2.34	0.65767	148	0.17752	322	9.51984	174	9.74899	146	0.82171	167	1.59817	325
2.36	0.65915	147	0.18074	319	9.52158	172	9.74753	146	0.82338	166	1.60142	322
2.38	0.66062	145	0.18393	315	9.52330	171	9.74607	146	0.82504	164	1.60464	319
2.40	0.66207	144	0.18708	312	9.52501	168	9.74461	145	0.82668	163	1.60783	317
2.42	0.66351	143	0.19020	309	9.52669	166	9.74316	145	0.82831	161	1.61100	314
2.44	0.66494	141	0.19329	306	9.52835	164	9.74171	144	0.82992	160	1.61414	311
2.46	0.66635	140	0.19635	302	9.52999	162	9.74027	144	0.83152	158	1.61725	308
2.48	0.66775	139	0.19937	299	9.53161	161	9.73883	143	0.83310	157	1.62033	305
2.50	0.66914	137	0.20236	296	9.53322	159	9.73740	143	0.83467	156	1.62338	302
2.52	0.67051	136	0.20532	293	9.53481	157	9.73597	142	0.83623	154	1.62640	300
2.54	0.67187	135	0.20825	290	9.53638	155	9.73455	142	0.83777	153	1.62940	297
2.56	0.67322	134	0.21115	287	9.53793	154	9.73313	141	0.83930	152	1.63237	295
2.58	0.67456	133	0.21402	285	9.53947	151	9.73172	141	0.84082	150	1.63532	292
2.60	0.67589	132	0.21687	282	9.54098	150	9.73031	140	0.84232	149	1.63824	290
2.62	0.67721	131	0.21969	279	9.54248	148	9.72891	140	0.84381	148	1.64114	287

TABLE I—*Continued*

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
2.64	0.67852	130	0.22248	277	9.54396	147	9.72751	−140	0.84529	147	1.64401	285
2.66	0.67982	128	0.22525	274	9.54543	146	9.72611	139	0.84676	146	1.64686	283
2.68	0.68110	127	0.22799	271	9.54689	144	9.72472	139	0.84822	144	1.64969	281
2.70	0.68237	126	0.23070	269	9.54833	143	9.72333	138	0.84966	144	1.65250	279
2.72	0.68363	125	0.23339	266	9.54976	141	9.72195	138	0.85110	143	1.65529	277
2.74	0.68488	125	0.23605	264	9.55117	140	9.72057	138	0.85253	141	1.65806	274
2.76	0.68613	124	0.23869	261	9.55257	138	9.71919	137	0.85394	140	1.66080	273
2.78	0.68737	122	0.24130	259	9.55395	136	9.71782	137	0.85534	139	1.66353	270
2.80	0.68859	121	0.24389	257	9.55531	135	9.71645	136	0.85673	138	1.66623	269
2.82	0.68980	121	0.24646	254	9.55666	134	9.71509	136	0.85811	137	1.66892	267
2.84	0.69101	119	0.24900	252	9.55800	133	9.71373	135	0.85948	136	1.67159	264
2.86	0.69220	119	0.25152	250	9.55933	131	9.71238	135	0.86084	135	1.67423	262
2.88	0.69339	118	0.25402	248	9.56064	130	9.71103	134	0.86219	134	1.67685	260
2.90	0.69457	117	0.25650	246	9.56194	129	9.70969	134	0.86353	133	1.67945	258
2.92	0.69574	116	0.25896	244	9.56323	128	9.70835	133	0.86486	132	1.68203	256
2.94	0.69690	115	0.26140	242	9.56451	126	9.70702	133	0.86618	131	1.68459	254
2.96	0.69805	114	0.26382	239	9.56577	125	9.70569	133	0.86749	130	1.68713	252
2.98	0.69919	113	0.26621	237	9.56702	124	9.70436	132	0.86879	130	1.68965	251
3.00	0.70032	280	0.26858	584	9.56826	304	9.70304	328	0.87009	319	1.69216	618
3.05	0.70312	275	0.27442	572	9.57130	297	9.66976	326	0.87328	314	1.69834	608
3.10	0.70587	270	0.28014	561	9.57427	290	9.69650	323	0.87642	308	1.70442	597
3.15	0.70857	265	0.28575	549	9.57717	284	9.69327	320	0.87950	302	1.71039	588
3.20	0.71122	261	0.29124	538	9.58001	278	9.69007	318	0.88252	297	1.71627	578
3.25	0.71383	256	0.29662	528	9.58279	272	9.68689	315	0.88549	293	1.72205	568
3.30	0.71639	252	0.30190	518	9.58551	266	9.68374	312	0.88842	289	1.72773	559

TABLE I—*Continued*

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
3.35	0.71891	249	0.30708	509	9.58817	260	9.68062	−310	0.89131	285	1.73332	550
3.40	0.72140	244	0.31217	499	9.59077	255	9.67752	307	0.89416	279	1.73882	541
3.45	0.72384	240	0.31716	489	9.59332	250	9.67445	305	0.89695	275	1.74423	533
3.50	0.72624	236	0.32205	481	9.59582	244	9.67140	302	0.89970	271	1.74956	525
3.55	0.72860	233	0.32686	473	9.59826	240	9.66838	300	0.90241	267	1.75481	513
3.60	0.73093	229	0.33159	464	9.60066	235	9.66538	297	0.90508	263	1.75994	508
3.65	0.73322	226	0.33623	456	9.60301	231	9.66241	295	0.90771	259	1.76502	502
3.70	0.73548	223	0.34079	449	9.60532	226	9.65946	293	0.91030	255	1.77004	496
3.75	0.73771	219	0.34528	441	9.60758	221	9.65653	290	0.91285	252	1.77500	489
3.80	0.73990	216	0.34969	433	9.60979	217	9.65363	288	0.91537	247	1.77989	482
3.85	0.74206	213	0.35402	427	9.61196	214	9.65075	285	0.91784	245	1.78471	474
3.90	0.74419	210	0.35829	420	9.61410	210	9.64790	283	0.92029	243	1.78945	467
3.95	0.74629	207	0.36249	413	9.61620	205	9.64507	282	0.92272	238	1.79412	460
4.00	0.74836	204	0.36662	406	9.61825	202	9.64225	279	0.92510	235	1.79872	454
4.05	0.75040	201	0.37068	400	9.62027	199	9.63946	277	0.92745	232	1.80326	449
4.10	0.75241	199	0.37468	394	9.62226	195	9.63669	275	0.92977	229	1.80775	443
4.15	0.75440	197	0.37862	388	9.62421	192	9.63394	272	0.93206	226	1.81218	438
4.20	0.75637	194	0.38250	382	9.62613	188	9.63122	270	0.93432	223	1.81656	431
4.25	0.75831	191	0.38632	376	9.62801	186	9.62852	268	0.93655	220	1.82087	426
4.30	0.76022	189	0.39008	371	9.62987	182	9.62584	267	0.93875	218	1.82513	421
4.35	0.76211	187	0.39379	366	9.63169	179	9.62317	264	0.94093	215	1.82934	415
4.40	0.76398	184	0.39745	360	9.63348	176	9.62053	262	0.94308	212	1.83349	410
4.45	0.76582	182	0.40105	355	9.63524	173	9.61791	260	0.94520	210	1.83759	406
4.50	0.76764	180	0.40460	351	9.63697	171	9.61531	259	0.94730	208	1.84165	401
4.55	0.76944	177	0.40811	346	9.63868	168	9.61272	257	0.94938	205	1.84566	396

TABLE I—*Continued*

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
4.60	0.77121	175	0.41157	340	9.64036	165	9.61015	−255	0.95143	202	1.84962	391
4.65	0.77296	174	0.41497	336	9.64201	163	9.60760	253	0.95345	200	1.85353	387
4.70	0.77470	171	0.41833	332	9.64364	160	9.60507	250	0.95545	198	1.85740	382
4.75	0.77641	169	0.42165	327	9.64524	158	9.60257	249	0.95743	196	1.86122	378
4.80	0.77810	167	0.42492	323	9.64682	155	9.60008	247	0.95939	193	1.86500	374
4.85	0.77977	166	0.42815	319	9.64837	153	9.59761	246	0.96132	191	1.86874	370
4.90	0.78143	164	0.43134	314	9.64990	151	9.59515	244	0.96323	189	1.87244	366
4.95	0.78307	162	0.43448	311	9.65141	149	9.59271	242	0.96512	188	1.87610	361
5.00	0.78469	160	0.43759	306	9.65290	146	9.59029	240	0.96700	185	1.87971	357
5.05	0.78629	158	0.44065	303	9.65436	145	9.58789	238	0.96885	183	1.88328	354
5.10	0.78787	157	0.44368	299	9.65581	142	9.58551	237	0.97068	182	1.88682	350
5.15	0.78944	155	0.44667	296	9.65723	141	9.58314	235	0.97250	180	1.89032	347
5.20	0.79099	153	0.44963	292	9.65864	138	9.58079	234	0.97430	177	1.89379	343
5.25	0.79252	152	0.45255	288	9.66002	137	9.57845	232	0.97607	176	1.89722	339
5.30	0.79404	150	0.45543	285	9.66139	135	9.57613	231	0.97783	174	1.90061	336
5.35	0.79554	149	0.45828	282	9.66274	133	9.57382	229	0.97957	173	1.90397	333
5.40	0.79703	147	0.46110	278	9.66407	131	9.57153	228	0.98130	171	1.90730	329
5.45	0.79850	146	0.46388	275	9.66538	129	9.56925	225	0.98301	169	1.91059	326
5.50	0.79996	145	0.46663	273	9.66667	128	9.56700	225	0.98470	167	1.91385	323
5.55	0.80141	143	0.46936	269	9.66795	126	9.56475	223	0.98637	166	1.91708	320
5.60	0.80284	142	0.47205	266	9.66921	124	9.56252	221	0.98803	164	1.92028	317
5.65	0.80426	140	0.47471	263	9.67045	123	9.56031	220	0.98967	162	1.92345	314
5.70	0.80566	139	0.47734	260	9.67168	121	9.55811	219	0.99129	161	1.92659	311
5.75	0.80705	137	0.47994	257	9.67289	120	9.55592	217	0.99290	160	1.92970	307
5.80	0.80842	136	0.48251	254	9.67409	118	9.55375	215	0.99450	158	1.93277	304

TABLE I—*Continued*

x	$\log X_0$	D	$\log X_1$	D	$\log X_2$	D	$\log X_3$	D	$\log X_4$	D	$\log X_5$	D
5.85	0.80978	135	0.48505	252	9.67527	117	9.55160	−214	0.99608	157	1.93581	302
5.90	0.81113	134	0.48757	249	9.67644	116	9.54946	213	0.99765	155	1.93883	299
5.95	0.81247	132	0.49006	247	9.67760	114	9.54733	212	0.99920	154	1.94182	297
6.00	0.81379	262	0.49253	485	9.67874	224	9.54521	419	1.00074	304	1.94479	586
6.1	0.81641	256	0.49738	475	9.68098	218	9.54102	415	1.00378	298	1.95065	575
6.2	0.81897	252	0.50213	466	9.68316	213	9.53687	408	1.00676	293	1.95640	565
6.3	0.82149	248	0.50679	457	9.68529	209	9.53279	405	1.00969	288	1.96205	555
6.4	0.82397	244	0.51136	447	9.68738	204	9.52874	399	1.01257	283	1.96760	546
6.5	0.82641	240	0.51583	439	9.68942	200	9.52475	394	1.01540	279	1.97306	538
6.6	0.82881	236	0.52022	430	9.69142	195	9.52081	390	1.01819	274	1.97844	528
6.7	0.83117	232	0.52452	423	9.69337	191	9.51691	385	1.02093	270	1.98372	519
6.8	0.83349	228	0.52875	415	9.69528	186	9.51306	381	1.02363	265	1.98891	511
6.9	0.83577	224	0.53290	408	9.69714	183	9.50925	376	1.02628	262	1.99402	503
7.0	0.83801	222	0.53698	400	9.69897	179	9.50549	372	1.02890	258	1.99905	497
7.1	0.84023	218	0.54098	394	9.60076	176	9.50177	368	1.03148	254	2.00402	490
7.2	0.84241	215	0.54492	387	9.70252	172	9.49809	364	1.03402	250	2.00892	481
7.3	0.84456	211	0.54879	380	9.70424	168	9.49445	360	1.03652	246	2.01373	474
7.4	0.84667	208	0.55259	373	9.70592	165	9.49085	356	1.03898	242	2.01847	468
7.5	0.84875	206	0.55632	368	9.70757	162	9.48729	352	1.04140	239	2.02315	461
7.6	0.85081	203	0.56000	361	9.70919	159	9.48377	349	1.04379	236	2.02776	454
7.7	0.85284	199	0.56361	356	9.71078	156	9.48028	345	1.04615	233	2.03230	447
7.8	0.85483	196	0.56717	349	9.71234	153	9.47683	341	1.04848	230	2.03677	440
7.9	0.85679	194	0.57066	345	9.71387	151	9.47342	338	1.05078	226	2.04117	435
8.0	0.85873	192	0.57411	339	9.71538	147	9.47004	333	1.05304	223	2.04552	431
8.1	0.86065	189	0.57750	334	9.71685	145	9.46671	330	1.05527	221	2.04983	425

TABLE I—*Continued*

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
8.2	0.86254	187	0.58084	329	9.71830	142	9.46341	− 328	1.05748	217	2.05408	419
8.3	0.86441	184	0.58413	324	9.71972	140	9.46013	324	1.05965	215	2.05827	413
8.4	0.86625	182	0.58737	319	0.72112	138	9.45689	321	1.06180	212	2.06240	407
8.5	0.86807	179	0.59056	315	9.72250	135	9.45368	318	1.06392	209	2.06647	403
8.6	0.86986	177	0.59371	310	9.72385	133	9.45050	314	1.06601	207	2.07050	398
8.7	0.87163	175	0.59681	305	9.72518	130	9.44736	312	1.06808	204	2.07448	393
8.8	0.87338	173	0.59986	301	9.72648	128	9.44424	309	1.07012	202	2.07841	388
8.9	0.87511	171	0.60287	298	9.72776	127	9.44115	306	1.07214	199	2.08229	383
9.0	0.87682	169	0.60585	293	9.72903	124	9.43809	303	1.07413	196	2.08612	379
9.1	0.87851	166	0.60878	289	9.73027	123	9.43506	300	1.07609	195	2.08991	374
9.2	0.88017	165	0.61167	285	9.73150	121	9.43206	297	1.07804	193	2.09365	369
9.3	0.88182	163	0.61452	282	9.73271	119	9.42909	295	1.07997	190	2.09734	366
9.4	0.88345	161	0.61734	278	9.73390	116	9.42614	292	1.08187	187	2.10100	362
9.5	0.88506	159	0.62012	274	9.73506	115	9.42322	289	1.08374	186	2.10462	357
9.6	0.88665	157	0.62286	271	9.73621	113	9.42033	287	1.08560	184	2.10819	353
9.7	0.88822	156	0.62557	267	9.73734	112	9.41746	284	1.08744	182	2.11172	350
9.8	0.88978	154	0.62824	264	9.73846	111	9.41462	282	1.08926	179	2.11522	345
9.9	0.89132	152	0.63088	261	9.73957	108	9.41180	279	1.09105	178	2.11867	342
10.0	0.89284	151	0.63349	257	9.74065	106	9.40901	277	1.09283	176	2.12209	338
10.1	0.89435	149	0.63606	254	9.74171	105	9.40624	275	1.09459	174	2.12547	335
10.2	0.89584	147	0.63860	251	9.74276	104	9.40349	272	1.09633	172	2.12882	331
10.3	0.89731	146	0.64111	249	9.74380	102	9.40077	270	1.09805	171	2.13213	327
10.4	0.89877	145	0.64360	246	9.74482	101	9.39807	268	1.09976	169	2.13540	324
10.5	0.90022	143	0.64606	242	9.74583	100	9.39539	265	1.10145	167	2.13864	322
10.6	0.90165	142	0.64848	239	9.74683	98	9.39274	263	1.10312	165	2.14186	318

TABLE I—*Continued*

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
10.7	0.90307	140	0.65087	237	9.74781	96	9.39011	−262	1.10477	163	2.14504	314
10.8	0.90447	139	0.65324	234	9.74877	95	9.38749	259	1.10640	162	2.14818	311
10.9	0.90586	137	0.65558	232	9.74972	95	9.38490	257	1.10802	161	2.15129	308
11.0	0.90723	135	0.65790	229	9.75067	93	9.38233	255	1.10963	159	2.15437	305
11.1	0.90858	135	0.66019	226	9.75160	92	9.37978	253	1.11122	157	2.15742	303
11.2	0.90993	134	0.66245	224	9.75252	91	9.37725	251	1.11279	156	2.16045	300
11.3	0.91127	132	0.66469	222	9.75343	89	9.37474	249	1.11435	154	2.16345	297
11.4	0.91259	131	0.66691	219	9.75432	88	9.37225	248	1.11589	153	2.16642	294
11.5	0.91390	130	0.66910	217	9.75520	87	9.36977	245	1.11742	151	2.16936	291
11.6	0.91520	129	0.67127	215	9.75607	86	9.36732	244	1.11893	150	2.17227	288
11.7	0.91649	127	0.67342	212	9.75693	85	9.36488	241	1.12043	149	2.17515	286
11.8	0.91776	126	0.67554	210	9.75778	84	9.36247	240	1.12192	147	2.17801	283
11.9	0.91902	125	0.67764	208	9.75862	83	9.36007	237	1.12339	146	2.18084	280
12.0	0.92027	124	0.67972	205	9.75945	82	9.35770	235	1.12485	144	2.18364	277
12.1	0.92151	123	0.68177	204	9.76027	81	9.35535	234	1.12629	143	2.18641	275
12.2	0.92274	122	0.68381	202	9.76108	80	9.35301	233	1.12772	143	2.18916	273
12.3	0.92396	120	0.68583	200	9.76188	79	9.35068	232	1.12915	142	2.19189	271
12.4	0.92516	119	0.68783	198	9.76267	78	9.34836	230	1.13057	140	2.19460	269
12.5	0.92635	119	0.68981	195	9.76345	77	9.34606	227	1.13197	138	2.19729	267
12.6	0.92754	118	0.69176	194	9.76422	77	9.34379	226	1.13335	138	2.19996	264
12.7	0.92872	117	0.69370	192	9.76499	75	9.34153	225	1.13473	136	2.20260	262
12.8	0.92989	115	0.69562	190	9.76574	74	9.33928	223	1.13609	134	2.20522	260
12.9	0.93104	115	0.69752	189	9.76648	74	9.33705	221	1.13743	134	2.20782	257
13.0	0.93219	114	0.69941	187	9.76722	73	9.33484	220	1.13877	133	2.21039	255
13.1	0.93333	113	0.70128	185	9.76795	72	9.33264	219	1.14010	132	2.21294	253

TABLE I—*Continued*

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
13.2	0.93446	112	0.70313	183	9.76867	71	9.33045	−217	1.14142	130	2.21547	251
13.3	0.93558	111	0.70496	182	9.76938	71	9.32828	215	1.14272	130	2.21798	249
13.4	0.93669	110	0.70678	180	9.77009	70	9.32613	214	1.14402	129	2.22047	247
13.5	0.93779	109	0.70858	178	0.77079	69	9.32399	213	1.14531	128	2.22294	245
13.6	0.93888	108	0.71036	177	9.77148	68	9.32186	211	1.14659	127	2.22539	243
13.7	0.93996	108	0.71213	175	9.77216	68	9.31975	209	1.14786	125	2.22782	241
13.8	0.94104	107	0.71388	174	9.77284	67	9.31766	208	1.14911	124	2.23023	239
13.9	0.94211	106	0.71562	172	9.77351	66	9.31558	208	1.15035	124	2.23262	238
14.0	0.94317	210	0.71734	340	9.77417	130	9.31350	410	1.15159	244	2.23500	470
14.2	0.94527	206	0.72074	334	9.77547	128	9.30940	405	1.15403	241	2.23970	463
14.4	0.94733	203	0.72408	328	9.77675	125	9.30535	339	1.15644	238	2.24433	455
14.6	0.94936	201	0.72736	323	9.77800	122	9.30136	395	1.15882	233	2.24888	449
14.8	0.95137	197	0.73059	318	9.77922	121	9.29741	390	1.16115	231	2.25337	443
15.0	0.95334	195	0.73377	312	9.78043	117	9.29351	385	1.16346	227	2.25780	436
15.2	0.95529	192	0.73689	308	9.78160	116	9.28966	381	1.16573	224	2.26216	431
15.4	0.95721	189	0.73997	304	9.78276	115	9.28585	377	1.16797	221	2.26647	426
15.6	0.95910	187	0.74301	298	9.78391	110	9.28208	371	1.17018	218	2.27073	422
15.8	0.96097	185	0.74599	293	9.78501	109	9.27837	367	1.17236	214	2.27495	408
16.0	0.96282	181	0.74892	289	9.78610	108	9.27470	363	1.17450	213	2.27903	406
16.2	0.96463	180	0.75181	285	9.78718	105	9.27107	359	1.17663	209	2.28309	402
16.4	0.96643	177	0.75466	281	9.78823	104	9.26748	355	1.17872	206	2.28711	397
16.6	0.96820	175	0.75747	277	9.78927	102	9.26393	351	1.18078	204	2.29108	392
16.8	0.96995	173	0.76024	273	9.79029	100	9.26042	347	1.18282	201	2.29500	386
17.0	0.97168	170	0.76297	269	9.79129	98	9.25695	343	1.18483	199	2.29886	382
17.2	0.97338	169	0.76566	265	9.79227	97	9.25352	340	1.18682	197	2.30268	377

TABLE I—*Continued*

x	$\log X_0$	D	$\log X_1$	D	$\log X_2$	D	$\log X_3$	D	$\log X_4$	D	$\log X_5$	D
17.4	0.97507	166	0.76831	262	9.79324	95	9.25012	−336	1.18879	193	2.30645	372
17.6	0.97673	165	0.77093	258	9.79419	94	9.24676	332	1.19072	192	2.31017	368
17.8	0.97838	163	0.77351	255	9.79513	92	9.24344	329	1.19264	190	2.31385	365
18.0	0.98001	160	0.77606	250	9.79605	91	9.24015	326	1.19454	186	2.31750	358
18.2	0.98161	159	0.77856	248	9.79696	89	9.23689	322	1.19640	185	2.32108	355
18.4	0.98320	157	0.78104	245	9.79785	87	9.23367	319	1.19825	183	2.32463	351
18.6	0.98477	155	0.78349	242	9.79872	87	9.23048	316	1.20008	180	2.32814	347
18.8	0.98632	153	0.78591	238	9.79959	85	9.22732	313	1.20188	179	2.33161	343
19.0	0.98785	152	0.78829	235	9.80044	84	9.22419	310	1.20367	176	2.33504	339
19.2	0.98937	149	0.79064	232	9.80128	82	9.22109	306	1.20543	174	2.33843	334
19.4	0.99086	149	0.79296	230	9.80210	82	9.21803	304	1.20717	174	2.34177	333
19.6	0.99235	147	0.79526	228	9.80292	80	9.21499	301	1.20891	171	2.34510	328
19.8	0.99382	145	0.79754	225	9.80372	79	9.21198	298	1.21062	168	2.34838	326
20.0	0.99527	145	0.79979	222	9.80451	77	9.20900	295	1.21230	175	2.35164	326
20.2	0.99672	145	0.80201	220	9.80528	76	9.20605	288	1.21405	167	2.35490	323
20.4	0.99817	144	0.80421	219	9.80604	75	9.20317	295	1.21572	163	2.35813	318
20.6	0.99961	143	0.80640	218	9.80679	75	9.20022	291	1.21735	161	2.36131	315
20.8	1.00104	143	0.80858	217	9.80754	74	9.19731	288	1.21896	159	2.36446	310
21.0	1.00247	142	0.81075	215	9.80828	73	9.19443	287	1.22055	159	2.36756	306
21.2	1.00389	138	0.81290	209	9.80901	71	9.19156	286	1.22214	159	2.37062	303
21.4	1.00527	137	0.81499	208	9.80972	71	9.18870	276	1.22373	155	2.37365	298
21.6	1.00664	136	0.81707	206	9.81043	70	9.18594	273	1.22528	155	2.37663	295
21.8	1.00800	137	0.81913	206	9.81113	69	9.18321	268	1.22683	152	2.37958	291
22.0	1.00937	131	0.82119	199	9.81182	68	9.18053	265	1.22835	149	2.38249	287
22.2	1.01068	131	0.82318	199	9.81250	68	9.17788	263	1.22984	145	2.38536	283

TABLE I—*Continued*

x	log X_0	D	log X_1	D	log X_2	D	log X_3	D	log X_4	D	log X_5	D
22.4	1.01199	127	0.82517	193	9.81318	66	9.17525	262	1.23129	145	2.38819	282
22.6	1.01326	124	0.82710	191	9.81384	65	9.17263	264	1.23274	145	2.39101	278
22.8	1.01450	123	0.82901	186	9.81449	64	9.16999	264	1.23419	144	2.39379	276
23.0	1.01573	122	0.83087	185	9.81513	64	9.16735	261	1.23563	141	2.39655	274
23.2	1.01695	117	0.83272	180	9.81577	63	9.16474	256	1.23704	141	2.39929	273
23.4	1.01812	116	0.83452	178	9.81640	62	9.16218	252	1.23845	140	2.40202	271
23.6	1.01928	116	0.83630	177	9.81702	61	9.15966	248	1.23985	140	2.40473	270
23.8	1.02044	116	0.83807	176	9.81763	60	9.15718	245	1.24125	139	2.40743	267
24.0	1.02160	116	0.83983	176	9.81823	60	9.15473	244	1.24264	139	2.41010	265
24.2	1.02276	115	0.84159	174	9.81883	59	9.15229	239	1.24403	138	2.41275	262
24.4	1.02391	115	0.84333	174	9.81942	59	9.14990	235	1.24541	138	2.41537	262
24.6	1.02506	115	0.84507	173	9.82001	58	9.14755	229	1.24679	138	2.41799	259
24.8	1.02621	114	0.84680	171	9.82059	57	9.14526	231	1.24817	137	2.42058	257
25.0	1.02735	114	0.84851	170	9.82116	57	9.14295	228	1.24954	137	2.42315	255

SUPPLEMENT TO TABLE I

x	X_0	D	X_1	D	X_2	D	X_3	D	X_4	D
0.1	1.0838	4336	0.0339	556	0.0313	277	0.4841	1276	1.46	59
0.2	1.5174	3238	0.0895	647	0.0590	247	0.6117	638	2.05	45
0.3	1.8412	2661	0.1542	694	0.0837	224	0.6755	333	2.50	38
0.4	2.1073	2191	0.2236	718	0.1061	203	0.7088	151	2.88	33
0.5	2.3364	2028	0.2954	728	0.1264	186	0.7249	+ 58	3.21	30
0.6	2.5392	1828	0.3682	731	0.1450	171	0.7307	− 8	3.51	27
0.7	2.7220	1670	0.4413	727	0.1621	158	0.7299	50	3.78	25
0.8	2.8890	1543	0.5140	722	0.1779	147	0.7249	77	4.03	23
0.9	3.0433	1434	0.5862	712	0.1926	137	0.7172	95	4.26	22
1.0	3.1867	1344	0.6574	703	0.2063	128	0.7077	106	4.48	20
1.1	3.3211	1265	0.7277	691	0.2191	120	0.6971	112	4.68	20
1.2	3.4476	1197	0.7968	680	0.2311	113	0.6859	117	4.88	18
1.3	3.5673	1137	0.8648	669	0.2424	107	0.6742	117	5.06	18
1.4	3.6810	1083	0.9317	656	0.2531	101	0.6625	117	5.24	17
1.5	3.7893	1035	0.9973	645	0.2632	96	0.6508	116	5.41	17
1.6	3.8928	992	1.0618	634	0.2728	91	0.6392	115	5.58	15
1.7	3.9920	952	1.1252	622	0.2819	86	0.6277	112	5.73	16
1.8	4.0872	917	1.1874	611	0.2905	83	0.6165	109	5.89	14
1.9	4.1789	884	1.2485	600	0.2988	78	0.6056	106	6.03	15
2.0	4.2673	853	1.3085	590	0.3066	76	0.5950	104	6.18	14
2.1	4.3526	826	1.3675	579	0.3142	72	0.5846	100	6.32	13

SUPPLEMENT TO TABLE I—*Continued*

x	X_0	D	X_1	D	X_2	D	X_3	D	X_4	D
2.2	4.4352	799	1.4254	570	0.3214	69	0.5746	− 98	6.45	13
2.3	4.5151	776	1.4824	560	0.3283	67	0.5648	94	6.58	13
2.4	4.5927	754	1.5384	551	0.3350	64	0.5554	91	6.71	12
2.5	4.6681	731	1.5935	542	0.3414	61	0.5463	89	6.83	13
2.6	4.7412	713	1.6477	533	0.3475	60	0.5374	86	6.96	11
2.7	4.8125	694	1.7010	524	0.3535	57	0.5288	83	7.07	12
2.8	4.8819	677	1.7534	517	0.3592	55	0.5205	80	7.19	11
2.9	4.9496	660	1.8051	509	0.3647	53	0.5125	78	7.30	11
3.0	5.0156	645	1.8560	501	0.3700	52	0.5047	75	7.41	11
3.1	5.0801	629	1.9061	493	0.3752	50	0.4972	73	7.52	11
3.2	5.1430	616	1.9554	486	0.3802	48	0.4899	71	7.63	10
3.3	5.2046	604	2.0040	480	0.3850	47	0.4828	69	7.73	..
3.4	5.2650	590	2.0520	472	0.3897	46	0.4759	67	7.84	10
3.5	5.3240	578	2.0992	466	0.3943	44	0.4692	64	7.94	10
3.6	5.3818	567	2.1458	460	0.3987	43	0.4628	63	8.04	9
3.7	5.4385	557	2.1918	453	0.4030	42	0.4565	61	8.13	10
3.8	5.4942	545	2.2371	448	0.4072	40	0.4504	59	8.23	9
3.9	5.5487	535	2.2819	442	0.4112	40	0.4445	57	8.32	10
4.0	5.6022	1043	2.3261	866	0.4152	76	0.4388	110	8.42	18
4.2	5.7065	1009	2.4127	845	0.4228	72	0.4278	104	8.60	17
4.4	5.8074	975	2.4972	825	0.4300	69	0.4174	99	8.77	17

SUPPLEMENT TO TABLE I—*Continued*

x	X_0	D	X_1	D	X_2	D	X_3	D	X_4	D
4.6	5.9049	944	2.5797	805	0.4369	65	0.4075	−93	8.94	17
4.8	5.9993	917	2.6602	788	0.4434	63	0.3982	89	9.11	16
5.0	6.0910	890	2.7390	770	0.4497	60	0.3893	84	9.27	16
5.2	6.1800	866	2.8160	754	0.4557	57	0.3809	81	9.43	15
5.4	6.2666	844	2.8914	738	0.4614	55	0.3728	76	9.58	15
5.6	6.3510	821	2.9652	723	0.4669	53	0.3652	73	9.73	14
5.8	6.4331	801	3.0375	709	0.4722	50	0.3579	70	9.87	15
6.0	6.5132	781	3.1084	694	0.4772	49	0.3509	67	10.02	14
6.2	6.5913	763	3.1778	683	0.4821	47	0.3442	63	10.16	13
6.4	6.6676	748	3.2461	669	0.4868	46	0.3379	62	10.29	14
6.6	6.7424	730	3.3130	657	0.4914	44	0.3317	58	10.43	13
6.8	6.8154	713	3.3787	647	0.4958	42	0.3259	56	10.56	13
7.0	6.8867	701	3.4434	635	0.5000	41	0.3203	55	10.69	12
7.2	6.9568	686	3.5069	625	0.5041	40	0.3148	52	10.81	13
7.4	7.0254	673	3.5694	614	0.5081	38	0.3096	50	10.94	12
7.6	7.0927	660	3.6308	604	0.5119	37	0.3046	48	11.06	12
7.8	7.1587	645	3.6912	595	0.5156	37	0.2998	47	11.18	12
8.0	7.2232	637	3.7507	586	0.5193	35	0.2951	44	11.30	11
8.2	7.2869	625	3.8093	577	0.5228	34	0.2907	44	11.41	12
8.4	7.3494	613	3.8670	568	0.5262	33	0.2863	41	11.53	11
8.6	7.4107	603	3.9238	560	0.5295	32	0.2822	41	11.64	11

SUPPLEMENT TO TABLE I—*Continued*

x	X_0	D	X_1	D	X_2	D	X_3	D	X_4	D
8.8	7.4710	595	3.9798	553	0.5327	31	0.2781	−39	11.75	11
9.0	7.5305	1442	4.0351	1347	0.5358	75	0.2742	92	11.86	27
9.5	7.6747	1387	4.1698	1304	0.5433	71	0.2650	85	12.13	25
10.0	7.8134	1339	4.3002	1263	0.5504	66	0.2565	80	12.38	25
10.5	7.9473	1294	4.4265	1223	0.5570	62	0.2485	73	12.63	24
11.0	8.0767	1250	4.5488	1189	0.5632	59	0.2412	69	12.87	23
11.5	8.2017	1212	4.6677	1155	0.5691	56	0.2343	64	13.10	23
12.0	8.3229	1173	4.7832	1125	0.5747	53	0.2279	60	13.33	22
12.5	8.4402	1143	4.8957	1094	0.5800	51	0.2219	57	13.55	21
13.0	8.5545	1110	5.0051	1068	0.5851	48	0.2162	53	13.76	21
13.5	8.6655	1080	5.1119	1041	0.5899	46	0.2109	51	13.97	21
14.0	8.7735	1052	5.2160	1016	0.5945	45	0.2058	47	14.18	20
14.5	8.8787	1027	5.3176	995	0.5990	42	0.2011	45	14.38	19
15.0	8.9814	1002	5.4171	972	0.6032	40	0.1966	43	14.57	19
15.5	9.0816	980	5.5143	952	0.6072	39	0.1923	41	14.76	18
16.0	9.1796	954	5.6095	931	0.6111	37	0.1882	38	14.94	19
16.5	9.2750	938	5.7026	913	0.6148	36	0.1844	37	15.13	17
17.0	9.3688	916	5.7939	895	0.6184	35	0.1807	35	15.30	18
17.5	9.4604	898	5.8834	878	0.6219	33	0.1772	34	15.48	17
18.0	9.5502	878	5.9712	860	0.6252	33	0.1738	32	15.65	17
18.5	9.6380	862	6.0572	845	0.6285	31	0.1706	30	15.82	16

SUPPLEMENT TO TABLE I—*Continued*

x	X_0	D	X_1	D	X_2	D	X_3	D	X_4	D
19.0	9.7242	845	6.1417	831	0.6316	30	0.1676	−30	15.98	16
19.5	9.8087	830	6.2248	816	0.6346	29	0.1646	28	16.14	16
20.0	9.8917	816	6.3064	802	0.6375	28	0.1618	27	16.30	16
20.5	9.9733	880	6.3866	789	0.6403	28	0.1591	26	16.46	16
21.0	10.053	879	6.4655	777	0.6431	27	0.1565	26	16.62	15
21.5	10.132	877	6.5432	764	0.6458	26	0.1539	24	16.77	15
22.0	10.209	877	6.6196	751	0.6484	25	0.1515	23	16.92	14
22.5	10.286	875	6.6947	741	0.6509	24	0.1492	22	17.06	14
23.0	10.361	873	6.7688	730	0.6533	24	0.1470	22	17.20	14
23.5	10.434	873	6.8418	719	0.6557	23	0.1448	20	17.34	14
24.0	10.507	871	6.9137	709	0.6580	23	0.1428	19	17.48	14
24.5	10.578	871	6.9846	698	0.6603	22	0.1409	19	17.62	14
25.0	10.649	870	7.0544	688	0.6625	22	0.1390	18	17.76	14

TABLE II

$$K = 1 - (1 - k)^{\frac{1}{2}}$$

k	log K	D	k	log K	D	k	log K	D
0.60	9.56531	930	0.908	9.84304	206	0.979	9.93201	177
0.61	9.57461	922	0.910	9.84510	207	0.980	9.93378	181
0.62	9.58383	915	0.912	9.84717	209	0.981	9.93559	185
0.63	9.59298	908	0.914	9.84926	210	0.982	9.93744	189
0.64	9.60206	902	0.916	9.85136	212	0.983	9.93933	194
0.65	9.61108	896	0.918	9.85348	213	0.984	9.94127	199
0.66	9.62004	890	0.920	9.85561	215	0.985	9.94326	205
0.67	9.62894	886	0.922	9.85776	217	0.986	9.94531	211
0.68	9.63780	882	0.924	9.85993	218	0.987	9.94742	219
0.69	9.64662	879	0.926	9.86211	221	0.988	9.94961	227
0.70	9.65541	875	0.928	9.86432	222	0.989	9.95188	236
0.71	9.66416	872	0.930	9.86654	224	0.990	9.95424	122
0.72	9.67288	871	0.932	9.86878	226	0.9905	9.95546	125
0.73	9.68159	869	0.934	9.87104	229	0.9910	9.95671	128
0.74	9.69028	869	0.936	9.87333	231	0.9915	9.95799	132
0.75	9.69897	869	0.938	9.87564	234	0.9920	9.95931	135
0.76	9.70766	869	0.940	9.87798	236	0.9925	9.96066	139
0.77	9.71635	871	0.942	9.88034	239	0.9930	9.96205	144
0.78	9.72506	873	0.944	9.88273	242	0.9935	9.96349	150
0.79	9.73379	877	0.946	9.88515	245	0.9940	9.96499	154
0.80	9.74256	880	0.948	9.88760	248	0.9945	9.96653	162
0.81	9.75136	886	0.950	9.89008	252	0.9950	9.96815	169
0.82	9.76022	893	0.952	9.89260	256	0.9955	9.96984	178
0.83	9.76915	900	0.954	9.89516	260	0.9960	9.97162	189
0.84	9.77815	454	0.956	9.89776	264	0.9965	9.97351	203
0.845	9.78269	456	0.958	9.90040	269	0.9970	9.97554	218
0.850	9.78725	459	0.960	9.90309	274	0.9975	9.97772	241
0.855	9.79184	462	0.962	9.90583	280	0.9980	9.98013	272
0.860	9.79646	465	0.964	9.90863	285	0.9985	9.98285	319
0.865	9.80111	469	0.966	9.91148	292	0.9990	9.98604	414
0.870	9.80580	473	0.968	9.91440	300	0.9995	9.99018	982
0.875	9.81053	478	0.970	9.91740	152	1.0000	0.00000	. . .
0.880	9.81530	482	0.971	9.91892	155			
0.885	9.82012	488	0.972	9.92047	157			
0.890	9.82500	492	0.973	9.92204	160			
0.895	9.82992	500	0.974	9.92364	161			
0.900	9.83492	201	0.975	9.92525	165			
0.902	9.83693	202	0.976	9.92690	167			
0.904	9.83895	204	0.977	9.92857	170			
0.906	9.84099	205	0.978	9.93027	174			

TABLE III.—Giving the total work that dry gunpowder of the W. A. standard is capable of performing in the bore of a gun, in foot-tons per lb. of powder burned.[1]

Number of volumes of expansion.	Corresponding density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.	Number of volumes of expansion.	Corresponding density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.
1.00	1.000	....	....	1.56	.641	34.500	.819
1.01	.990	.980	.980	1.58	.633	35.301	.801
1.02	.980	1.936	.956	1.60	.625	36.086	.785
1.03	.971	2.870	.934	1.62	.617	36.855	.769
1.04	.962	3.782	.912	1.64	.610	37.608	.753
1.05	.952	4.674	.892	1.66	.602	38.346	.738
1.06	.943	5.547	.873	1.68	.595	39.069	.723
1.07	.935	6.399	.852	1.70	.588	39.778	.709
1.08	.926	7.234	.835	1.72	.581	40.474	.696
1.09	.917	8.051	.817	1.74	.575	41.156	.682
1.10	.909	8.852	.810	1.76	.568	41.827	.671
1.11	.901	9.637	.785	1.78	.562	42.486	.659
1.12	.893	10.406	.769	1.80	.555	43.133	.647
1.13	.885	11.160	.754	1.82	.549	43.769	.636
1.14	.877	11.899	.739	1.84	.543	44.394	.625
1.15	.870	12.625	.726	1.86	.537	45.009	.615
1.16	.862	13.338	.713	1.88	.532	45.614	.605
1.17	.855	14.038	.700	1.90	.526	46.209	.595
1.18	.847	14.725	.687	1.92	.521	46.795	.586
1.19	.840	15.400	.675	1.94	.515	47.372	.577
1.20	.833	16.063	.663	1.96	.510	47.940	.568
1.21	.826	16.716	.653	1.98	.505	48.499	.559
1.22	.820	17.359	.643	2.00	.500	49.050	.551
1.23	.813	17.992	.633	2.05	.488	50.383	1.333
1.24	.806	18.614	.622	2.10	.476	51.673	1.290
1.25	.800	19.226	.612	2.15	.465	52.922	1.249
1.26	.794	19.828	.602	2.20	.454	54.132	1.210
1.27	.787	20.420	.592	2.25	.444	55.304	1.172
1.28	.781	21.001	.581	2.30	.435	56.439	1.135
1.29	.775	21.572	.571	2.35	.425	57.539	1.100
1.30	.769	22.133	.561	2.40	.417	58.605	1.066
1.32	.758	23.246	1.113	2.45	.408	59.639	1.034
1.34	.746	24.324	1.078	2.50	.400	60.642	1.003
1.36	.735	25.371	1.047	2.55	.392	61.616	.974
1.38	.725	26.389	1.018	2.60	.384	62.563	.947
1.40	.714	27.380	.991	2.65	.377	63.486	.923
1.42	.704	28.348	.968	2.70	.370	64.385	.899
1.44	.694	29.291	.943	2.75	.363	65.262	.877
1.46	.685	30.211	.920	2.80	.357	66.119	.857
1.48	.676	31.109	.898	2.85	.351	66.955	.836
1.50	.667	31.986	.877	2.90	.345	67.771	.816
1.52	.658	32.843	.857	2.95	.339	68.568	.797
1.54	.649	33.681	.838	3.00	.333	69.347	.779

[1] From Noble and Abel's "Researches on Fired Gunpowder."

Number of volumes of expansion.	Corresponding density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.	Number of volumes of expansion.	Corresponding density of products of combustion.	Total work per lb. burned in foot-tons.	Difference.
3.05	.328	70.109	.762	7.10	.141	105.125	.539
3.10	.322	70.854	.745	7.20	.139	105.655	.530
3.15	.317	71.584	.731	7.30	.137	106.176	.521
3.20	.312	72.301	.716	7.40	.135	106.688	.512
3.25	.308	73.002	.701	7.50	.133	107.192	.504
3.30	.303	73.690	.688	7.60	.131	107.688	.496
3.35	.298	74.365	.675	7.70	.130	108.177	.489
3.40	.294	75.027	.662	7.80	.128	108.659	.482
3.45	.290	75.677	.650	7.90	.126	109.133	.474
3.50	.286	76.315	.638	8.00	.125	109.600	.467
3.55	.282	76.940	.625	8.10	.123	110.060	.460
3.60	.278	77.553	.613	8.20	.122	110.514	.454
3.65	.274	78.156	.603	8.30	.120	110.962	.448
3.70	.270	78.749	.593	8.40	.119	111.404	.442
3.75	.266	79.332	.583	8.50	.117	111.840	.436
3.80	.263	79.905	.573	8.60	.116	112.270	.430
3.85	.260	80.469	.564	8.70	.115	112.695	.425
3.90	.256	81.024	.555	8.80	.114	113.114	.419
3.95	.253	81.570	.546	8.90	.112	113.528	.414
4.00	.250	82.107	.537	9.00	.111	113.937	.409
4.10	.244	83.157	1.050	9.10	.110	114.341	.404
4.20	.238	84.176	1.019	9.20	.109	114.739	.398
4.30	.232	85.166	.990	9.30	.108	115.133	.394
4.40	.227	86.128	.962	9.40	.106	115.521	.388
4.50	.222	87.064	.936	9.50	.105	115.905	.384
4.60	.217	87.975	.911	9.60	.104	116.284	.379
4.70	.213	88.861	.886	9.70	.103	116.659	.375
4.80	.208	89.724	.863	9.80	.102	117.029	.370
4.90	.204	90.565	.841	9.90	.101	117.395	.366
5.00	.200	91.385	.820	10	.100	117.757	.362
5.10	.196	92.186	.801	11	.091	121.165	3.408
5.20	.192	92.968	.782	12	.083	124.239	3.074
5.30	.188	93.732	.764	13	.077	127.036	2.797
5.40	.185	94.479	.747	14	.071	129.602	2.566
5.50	.182	95.210	.731	15	.066	131.970	2.368
5.60	.178	95.925	.715	16	.062	134.168	2.198
5.70	.175	96.625	.700	17	.059	136.218	2.050
5.80	.172	97.310	.685	18	.055	138.138	1.920
5.90	.169	97.981	.671	19	.052	139.944	1.806
6.00	.166	98.638	.657	20	.050	141.647	1.703
6.10	.164	99.282	.644	21	.047	143.258	1.611
6.20	.161	99.915	.633	22	.045	144.788	1.530
6.30	.159	100.536	.621	23	.043	146.242	1.454
6.40	.156	101.145	.609	24	.042	147.629	1.387
6.50	.154	101.744	.599	25	.040	148.953	1.324
6.60	.151	102.333	.589	30	.033	154.800	5.847
6.70	.149	102.912	.579	35	.028	159.667	4.867
6.80	.147	103.480	.568	40	.025	163.828	4.161
6.90	.145	104.038	.558	45	.022	167.456	3.628
7.00	.143	104.586	.548	50	.020	170.671	3.215

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